THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXXVII APRIL 1913 NUMBER 3

THE WORK OF SIR WILLIAM HUGGINS

By GEORGE E. HALE

In these days of great observatories, completely equipped with powerful instruments and free from the handicaps of city smoke and glare, it is easy to underestimate the opportunities of the amateur, and the possibilities of research with limited instrumental means. An anonymous author in the *Edinburgh Review*, after expressing his doubts as to the value of some of the instruments of a new observatory, remarks: "Is there not in truth a double danger, that extravagance of scientific enterprise may defeat its own purpose by overproduction in one place, and killing moderate ambitions elsewhere?"[1] This is a fair question, which would seem to find an answer in the life and work of such a man as Sir William Huggins.

If we define an amateur as one who works because he cannot help it, Huggins may be classed in that great English group of amateurs which includes such men as Faraday, Darwin, and Rayleigh—men who have worked for no other motive than intense love for research, undeterred by obstacles or by dearth of instrumental means. Half a century ago large and expensive telescopes, though few in number, were not altogether lacking. In 1856, when Huggins acquired his first telescope, of 5 inches aperture,

[1] "The Tercentenary of the Telescope," *Edinburgh Review*, January 1910.

15-inch refractors were in use at Pulkowa and Harvard, and Lord Rosse's 6-foot reflector had yielded important discoveries at Parsonstown. Two years later Huggins purchased the 8-inch refractor with which he made his visual observations of stellar and nebular spectra. Lassell carried a 4-foot reflector to the favorable climate of Malta in 1863, but if this disturbed Huggins, who discovered the gaseous nature of the nebulae in the following year, we find no indication of the fact in his publications. On the contrary, we see him busy with a score of new projects, one of which, startling to the conservative mind of his day, was the establishment of a small physical and chemical laboratory as an adjunct to his observatory. In 1870 a 4-foot reflector, with modern equatorial mounting, was set up under the almost unexplored southern heavens at Melbourne. Huggins, in the same year, mounted an 18-inch mirror, borrowed from the Royal Society, on the smoke-obscured London hilltop where he lived and worked, with no increase of aperture, to the day of his death. The development of great refractors, and the return of the reflector to favor found him keenly alive to every advance and warmly sympathetic with those of his colleagues who were more favored than he. But there was no relaxation of effort or failure of interest; no discouragement because of limitations of instruments or of atmosphere. Discovery after discovery was communicated to the Royal Society, and the possibilities of further work in astrophysics suggested by every new development in physics or chemistry were discerned and eagerly debated. Indeed, he seemed to find in his modest equipment a strong incentive to greater effort, for in one of his presidential addresses to the Royal Society he remarks: "We must not forget that, in the case of institutions as well as of individuals, a powerful and healthy stimulus to the exertion needful for success arises from the necessity of coping with and overcoming difficulties, whether of a monetary or other kind."[1]

It is evident that Huggins would not have sympathized with the attitude of Sir James South. When Professor O. M. Mitchel visited him in 1842, South was involved in difficulties with a firm

[1] *The Royal Society*, p. 67.

of instrument makers, which had failed to complete the mounting of his large object-glass. Mitchel describes his visit as follows:[1]

One apartment was examined after another, until finally we reached a large room surmounted by a dome of great size and of an expensive construction, while fragments of the framework for mounting a great equatorial were scattered around.

"Here, sir," exclaimed Sir James, "you behold the wreck of all my hopes. Here I have expended thousands, and flattered myself that I was soon to possess the finest instrument in Europe; but it is all over, and there's an end."

I remarked that the object-glass was still in his possession, and might yet be mounted so as to realize his hopes and expectations.

"No," said Sir James, "Struve has reaped the golden harvest among the double stars, and there is little now for me to hope or expect."

It would be difficult to appreciate the feelings which at that moment were sweeping through the mind of the astronomer. Long-cherished visions of fame and high distinction, nay, perhaps of grand discoveries in the heavens, which for years had played round his hopes of the future, had fled forever. Another had reaped the golden harvest, and like Clairaut, who wept that there was not for him, as for Newton, the problem of the universe to solve, Sir James South could almost weep to think that another's eye had been permitted to sweep over the far-distant realms of space which he had long hoped might remain his own peculiar province.

Thirty years later, an American amateur, possessing only a 6-inch telescope, began a search for double stars in the city of Chicago. His days were spent in the law courts, his nights in sweeping the heavens, in the wake of many a telescope far larger than his own. But his labors were rewarded by the discovery with the 6-inch telescope of 451 new double stars, before he gave up his court duties to accept a position in a great observatory. Here, again, the man rose above his environment, and proved that devoted interest and true enthusiasm are more powerful than costly instruments.[2]

However, the fears of the Edinburgh reviewer must not be dismissed too lightly, for the point he raises is one of high impor-

[1] *Ormsby MacKnight Mitchel: Astronomer and General.* A biographical narrative. By his son, F. A. Mitchel, 1887.

[2] See the Introduction to *General Catalogue of 1290 Double Stars Discovered by S. W. Burnham*, where the South incident is quoted, and a list of Burnham's discoveries with various telescopes may be found.

tance. If it be true that modern observatories, with their expensive equipment, tend to discourage the serious amateur, then it may be doubted whether the best use is being made of the funds they represent. For the history of science teaches that original ideas and new methods, as well as great discoveries resulting from the patient accumulation of observations, frequently come from the amateur. To hinder his work in any serious way might conceivably do a greater injury than a large observatory could make good.

We may ask, however, whether conditions are materially different today from what they were in former times. There have always been observatories far transcending in equipment the resources of the isolated amateur or those of the average professional observer. But contributions from the minor instruments have continued to enrich astronomy, and if there are signs now of reduced activity they are certainly not conspicuous.

The reasons for the amateur's success should not be far to seek. Indeed, we see them most admirably illustrated in the career of Sir William Huggins. The purchase of his 8-inch refractor antedated by only a year the interpretation of the solar spectrum by Kirchhoff and Bunsen. Almost simultaneously Darwin published the *Origin of Species*, and the great controversy regarding evolution was begun. Thus a new method of analysis, which took no account of the distance of the luminous source, suddenly became available. A man of Sir William's imagination could not fail, while reflecting on this novel ally, to divine some of the less obvious applications of the spectroscope. Nor would he be without curiosity as to the bearing of the new method on the problem of inorganic evolution. Quick to appreciate his opportunity, he lost no time in applying the spectroscope to the analysis of starlight. Next he discovered in the gaseous clouds of the nebulae the raw material from which the stars may be evolved. And later, after a successful application of photography, he arranged his spectra in the probable order of stellar development.

This faculty to perceive new possibilities remained with Sir William to the end. No physicist of the new school showed greater enthusiasm over the rapid unfolding of the electron theory or the disclosures connected with radium. None watched with more

eagerness the discovery of new gases, or considered more carefully their bearing on astrophysical problems. When Runge found the D_3 line of helium double in his vacuum tube, Huggins instantly turned his spectroscope to the sun, though he had given up systematic solar work years before. Persisting in spite of bad weather and barely adequate instrumental means, he soon succeeded in detecting the duplicity of the solar line. So it was with every new advance, in whatever field. Sir William read of it at once, discussed it with Lady Huggins, called it to the attention of his correspondents, and carefully examined into its bearing on astrophysics. I have scores of letters and postcards illustrating this characteristic of his remarkably alert mind.

Thus we may explain the success of the founder of modern astrophysics. Discoveries like Kirchhoff's are almost unique, and few can ever have the opportunity of applying such an instrument as the spectroscope for the first time in a virgin field. But the chance of profiting by new advances always exists, and Huggins would have been a great investigator if his work had begun a quarter of a century later. Consider some of the numerous activities of his mind. In his earliest comparisons of terrestrial and stellar spectra (1863) he had in view the possibility of detecting displacements of lines indicating stellar motions, and four years later he made the first serious attempt in this direction.[1] The extraordinary developments of this department of stellar spectroscopy in the hands of Vogel, Keeler, Campbell, and others are too familiar to call for mention here. The laboratory work, begun by Sir William in 1863 for the purpose of interpreting stellar spectra,[2] was continued until 1905. In 1903 Sir William and Lady Huggins announced their remarkable discovery of the spectrum produced by the spontaneous luminous radiation of radium.[3] In 1870 Huggins had a large magnet constructed for the express purpose of detecting such modifications of spectral lines as he anticipated might be produced by a magnetic field.[4] Insufficient dispersion prevented the effect from being seen, and it was not until 1896 that it was first found by Zeeman. To Huggins we owe the first obser-

[1] *Scientific Papers*, p. 229.
[2] *Ibid*, p. 397.
[3] *Ibid.*, p. 444.
[4] *Ibid.*, p. 458.

vations of sun-spot spectra which distinguished between the different degrees of widening of different lines. He was the first to observe the form of a solar prominence through a widened slit, and his earliest attempts to use the spectroscope for this purpose were made in 1867, before the publication of the classic results of Janssen and Lockyer.[1]

One ordinarily thinks of Sir William in his home on Upper Tulse Hill, where, with the sole aid of his learned and devoted wife, he carried on his study and research. None who has entered that quiet library, its shelves overflowing with books, can ever forget the sight of the aged philosopher in his natural environment. But in spite of the fascination of his work, Huggins did not forget his obligations to the larger world of science. His broad interests and his years of activity in the Royal Society were recognized by his election to the presidency of that illustrious body in 1900. A quotation from one of his annual addresses on St. Andrew's Day will not be out of place here:

> On one central eminence, dominating alike the past, the present, and the future, science has for some years firmly entrenched herself—the position that through all the ages the cosmos has advanced, and is still advancing, by a process of orderly evolution. In the domain of the living the fact of progress by means of evolution was finally established by our illustrious countryman Darwin, and his prophet Huxley. In heredity and variation, the great discovery of Mendel, in the hands of one of our medallists of today, promises to bring the biologist nearer to his main quest, the fundamental nature of living things. In physics, only a few years ago Professor J. J. Thomson took by storm the outworks of the central citadel of Nature—the chemical atom. His later brilliant attacks, aided by the new artillery of radioactive radiations, may be said to have carried the keep itself. Material mass gives place to the electric mass of moving electric charges. On this view the chemical elements, each with its individual properties, but all falling into family groups according to a periodic law, have their origin in differences in the number of electrons and in the figures of their giddy dances, whirling within the atom. Material nature becomes simplified into electricity and ether—or, is it only ether? Passing from the atom to the heavens, within the memory of those living, science has taught us so to read the sunbeams and starbeams as to enable us to apply the methods of the laboratory to the heavenly bodies. By means of their radiations alone we discuss their chemical constitution and their orbital and other motions, which were before unknowable. By each discovery the

[1] *Scientific Papers*, p. 305.

vision of the world has become more glorious, the wonder of it more amazing, while chambers and palaces of Nature still unexplored remain the exhaustless heritage of all coming generations.[1]

In reading such addresses as this we appreciate why Huggins was so staunch a supporter of the Royal Society. He believed strongly in the specialized societies which, a century ago, began to develop from the older body as the need for them appeared. He argued for their continuance as separate organizations, and advocated their independence and complete autonomy. But he saw that the Royal Society was needed in his time quite as much as in the early days, when it included the whole of British science. Indeed, the very fact of specialization, with the consequent danger of narrow vision and limited range of interest, is a powerful argument for a society which insists on the unity of Nature. Moreover, the suggestive value in other branches of science of methods of procedure developed in a special field is perhaps greater now than ever before. Sometimes in its original form, but more often modified to suit different conditions, a new method of research may offer unlimited opportunities to the investigator. The wise student is always on the watch for such possibilities, and the weekly meetings of the Royal Society, where papers dealing with every department of science are presented, afford the best means of securing useful suggestions.

In *The Royal Society, or Science in the State and in the Schools*, Sir William has brought together his four addresses as president. The first of these, delivered in 1902, is entitled "Supreme Importance of Science to the Industries of the Country, Which Can Be Secured Only through Making Science an Essential Part of All Education." He perceived what the average British manufacturer has so strangely ignored: the great commercial importance of education in science. With the fruits of English discovery passing into the hands of the Germans, whose universities have so long fostered and spread abroad the spirit of research, the apathy of the British public is hard to understand. Huggins, speaking in plain language, pointed to the chief source of weakness—"the too

[1] "The Advisory Relation of the Royal Society to the State" (address delivered at the anniversary meeting, November 30, 1904), *The Royal Society*, p. 89.

close adherence of our older universities, and through them of our public schools, and all other schools in the country downward, to the traditional methods of teaching of mediaeval times." He pleaded for a decrease of examinational restrictions, and for the encouragement of research and independent reasoning on the part of the student. He contrasted the listlessness and lack of interest in their work which characterize many graduates under the existing system, with the enthusiastic devotion to knowledge for its own sake aroused in the student by observation and research. As a result of the public interest excited by this address, the Council of the Royal Society adopted a resolution urging the universities to make such modifications in their regulations as will "insure that a knowledge of science is recognized in schools and elsewhere as an essential part of general education."

The address of 1903 traced the rise of the specialized societies and discussed their relation to the Royal Society, while that of the following year dealt with "The Advisory Relation of the Royal Society to the State, and the Responsible Public Duties Which Rest Permanently upon the Society." Both of these furnish much food for thought to those who are concerned with the development of our own National Academy. They demonstrate beyond a doubt that the Royal Society is by no means a mere survival from a former state of science, but a great and ever-active force, occupying a place which no other body could fill.

The last address, showing the influence of science on the life and thought of the world, is on an equally high plane with the others. In it Sir William gives an eloquent account of the awakening of scientific thought during the reign of Queen Victoria. As he recounts the work of Darwin, we can easily understand how his mind was directed from the first toward the problem of stellar evolution, in the study of which he made his richest contributions to science.

Without attempting to outline the events of a long and active life, filled with successful research, we have glanced at two phases of Sir William's career. The one, dealing with his public services, forms an important chapter in the recent history of the Royal Society, which counted him as one of the ablest in its long line of

eminent leaders. The other includes his personal investigations, which lie at the base of astrophysics. The place he has left cannot be filled. With him passes the pioneer period of celestial spectroscopy. Qualitative tests, common to this and to all other branches of science in their early stages, gave way in his own and his colleagues' hands to the precise methods which now characterize spectroscopy. The term "astronomy of precision" no longer serves to differentiate the old astronomy from the new, for rigorous and exact measurement is equally common in both. Huggins contributed largely to this end, as from the outset he measured the lines of terrestrial, stellar, and nebular spectra as accurately as the means at his disposal permitted. Giving no evidence of advancing age, he moved forward into the second period of astrophysics, never losing the alert mind of the pioneer.

Every investigator may find useful and inspiring suggestions in the life and example of Sir William Huggins. Their surest message and strongest appeal will be to the amateur with limited instrumental means, and to the man, however situated, who would break new ground.

THE VISUAL AND PHOTOGRAPHIC RANGES AND THE PROVISIONAL ORBITS OF *Y PISCIUM* AND *RR DRACONIS*

By HARLOW SHAPLEY

A large majority of the eclipsing binaries whose light decreases more than one magnitude at principal minimum are composed of bright primary stars with larger faint companions. When the loss of light is greater than two magnitudes the faint companion is very often more than four times the volume of the bright component; and in such a case the luminosity per unit area of the former may be less than one-fiftieth that of the primary star, and is very rarely greater than one-twentieth. It is probable that the division of mass between the components of the average eclipsing system is not very unequal, and, therefore, that the greater volumes of the fainter components indicate relatively low densities. Assuming an equal division of the mass for each system and uniformly illuminated stellar disks, I have found from thirty-five systems of spectral classes B and A that the average density of the bright component (whose light determines the spectrum) is one-seventh that of the sun; while the average density of the twenty-nine faint components whose spectra are not known is less than one-thirtieth the solar density. If we consider the stellar disks darkened, according to the cosine law, to zero at the limb, rather than uniformly bright, the average density of the brighter component is reduced to about one-eleventh, but the average for the faint companions is only slightly increased. For the eclipsing systems of large range of variation the "equal mass" density of the faint components is often below this average; and, further, if we take into account the spectroscopic evidence which indicates, wherever the second spectrum can be measured, that the bright component is universally the more massive, we are led to believe in the extremely great rarity of these large, relatively dark companions and to assign them to a period earlier in the natural stellar evolution than is occupied by their denser primaries.

The determination of the secondary spectra in the systems which present deep eclipses, and hence possess great inequalities of surface intensities, is of importance, therefore, in the study of the development of the close binary systems. When the component stars differ in brightness more than one magnitude, the secondary spectrum is not discernible at maximum light. During the constant minimum phase of a total primary eclipse the light is wholly that of the faint companion, but, because of the faintness of the star at that time, the duration of the total obscuration of the bright star is too short, except perhaps in a few cases,[1] to permit the making of a spectrogram. It is possible, however, to estimate indirectly the color of the faint companion by determining photographically and visually the range of variation at the principal minimum. To be an effective test the eclipse must be total, or so nearly total that the light of the bright star which still remains at the deepest phase of the eclipse does not predominate over the light of the faint companion. There are many eclipsing stars suitable for an investigation of the difference between photographic and visual ranges. At present results have been obtained for *Y Piscium* and *RR Draconis*, two eclipsing binaries of exceptionally great range. The photographic curve of *Y Piscium* was determined at the Harvard College Observatory by Miss Cannon, who discovered the variable in 1911. One-half the observations during changing light are published in *Harvard Circular No. 165*. The remainder were made recently from multiple-exposure plates and have been generously furnished in manuscript by Professor E. C. Pickering. The photographic curve of *RR Draconis* was determined by Professor Seares with the 60-inch reflector at Mt. Wilson. His light-curve is discussed in detail, and compared with the partial visual curve obtained at the Laws Observatory, in *Contributions from the Mt. Wilson Solar Observatory*, No. 64. The paper has been kindly sent to me in manuscript in advance of publication. The visual curves I have determined with the sliding-prism polarizing photometer of the Princeton University Observatory.

Y Piscium.—The period determined by Miss Cannon in *Harvard Circular No. 165* is verified by the later photographic and visual

[1] Blažko, *Annales de l'Observatoire astronomique de Moscou*, II Série, **5**, 10, 1911.

observations, and but a small adjustment of the initial epoch is necessary. The elements of light-variation are, therefore,

$$\text{Min.} = \text{J.D. } 2410002.844 + 3^{d}.76582 \text{ E, G.M.T.}$$

The published photographic observations (not including those for which only an upper limit of the brightness was estimated) have been combined with the manuscript estimates into the normal points of the following table. The published observation of August 1, 1904, is rejected because of the abnormal residual, which probably

NORMAL PHOTOGRAPHIC MAGNITUDES OF Y *PISCIUM* DURING CHANGING LIGHT

Phase	Mag.	No. Obs.	O.−C.	Phase	Mag.	No. Obs.	O.−C.
$-0^{d}170$.....	9.30	2	+0.04	$+0^{d}064$.....	10.72	4	−0.03
.084.....	10.40	3	+ .07	.073.....	10.50	3	− .05
.069.....	10.63	3	− .01	.080.....	10.40	3	− .01
.042.....	11.53	3	+ .14	.100.....	10.07	3	+ .02
.025.....	11.83	3	− .10	.117.....	9.73	3	− .05
−0.005.....	12.30	3	− .07	.139.....	9.53	3	.00
+0.014.....	12.40	3	+ .14	+0.173.....	9.17	3	−0.06
+0.036.....	11.60	3	+0.01				

results from a defect in the photographic plate. The manuscript observations were made from plates secured August 21, September 1, 20, and October 24, 1911. The photographic brightness at maximum is $9^{mg}.00$, and since the star is of spectral class A the same value was adopted as the normal visual magnitude. The minimum is $12^{mg}.40$. The range of $3^{mg}.40$ means a loss of more than 95 per cent of the photographically effective light of the system, and though the minimum shows no constant phase, the eclipse must necessarily be nearly, if not exactly, total.

The light-curve is not sufficiently well defined by these observations to permit a definitive solution for the orbital elements, but it is satisfactory for the preliminary derivation of an orbit that will represent closely the actual conditions in the system. Following the method of Russell for solving a partial eclipse of uniformly illuminated disks,[1] I have first obtained from the intensity-curve which best represents the whole curve at principal eclipse one

[1] *Astrophysical Journal*, **35**, 326 ff., 1912.

relation between the ratio of the radii, k, and the percentage of the small star eclipsed at the deepest phase of minimum, a_0; namely,

$$\chi(k, a_0, \tfrac{1}{4}) = \frac{\sin^2\theta(\frac{1}{4})}{\sin^2\theta(\frac{1}{2})} = 1.77 ,$$

where $\theta(\frac{1}{4})$ and $\theta(\frac{1}{2})$ are the true longitudes in the orbit of the eclipsing body at the times when the loss of light is respectively one-fourth and one-half the total loss. A second numerical relation between these quantities could be secured from the equation

$$a_0 = 1 - \lambda_1 + \frac{1-\lambda_2}{k^2}$$

(where λ_1, λ_2 are the intensities at the two minima) if the secondary minimum had been observed, and without information concerning the range at both eclipses the problem is indeterminate. The maximum possible depth of the secondary (corresponding to central transit of equal stars) is less than $0^{mg}.05$, so its detection is beyond the accuracy of the observations. Because of the great range at primary, however, and because of the shape of the light-curve, the indeterminateness in this case lies between very narrow limits. Assuming different values for the depth of secondary minimum, $1-\lambda_2$, we can readily compute the possible values of k and a_0, using a table of the χ-function, and obtain the following results, which extend from the limiting and improbable case of a completely black companion to the other extreme of a grazing total eclipse.

$1-\lambda_2=0.000$,	0.004,	0.008,	0.013,	0.017
$a_0=0.956$,	0.97,	0.98,	0.99,	1.00
$k=0.59$,	0.595,	0.60,	0.61,	0.615

In all cases the large star is in front at principal eclipse. For the derivation of the remainder of the elements I have adopted as an example the mean of the above values, $k=0.60$, $a_0=0.98$; the secondary minimum is accordingly less than one-hundredth of a magnitude. In this instance, at principal minimum 2 per cent of the light of the small bright star remains; and since the maximum loss at that time is 0.956 of the normal light, the total light of the bright star, L_2, is 0.975, and L_1 is 0.025. Of the light remaining at minimum phase, four-sevenths is that of the faint component.

The surface intensity of the bright star (in photographic light) is approximately 100 times that of its companion; the inclination of the orbit to the tangent plane is $i=83^\circ.9$; the radius of the bright star in terms of the distance of centers is $r_2=0.144$, and of the faint one, $r_1=0.240$. The least apparent distance of centers is $\cos i=0.107$. Assuming the components equally massive, the density of the bright star is 0.16 of the solar density, and that of the faint one is 0.03.

It is useless to seek for more precise values of the elements from this curve, or even from one determined with the highest accuracy possible, without some knowledge of the distribution of brightness over the stellar disks. The above values are derived on the limiting hypothesis of uniform disks. We can go to the other limit and determine the elements on the assumption of disks completely darkened at the limb. The true values will lie between these two sets and will probably be closer to the "completely darkened" elements than to the "uniform" set just derived.[1] Using tables and equations especially suitable for this problem,[2] I have readily derived from the "uniform" elements the following values on the assumption that the disk of the brighter star is darkened to zero at the edge: $k=0.77$, $r_1=0.231$, $r_2=0.178$, $i=85^\circ.7$, $\cos i=0.075$, ratio of surface intensities 67 : 1, and the secondary minimum would be just less than $0^{mg}.02$. The values of a_0, L_1, L_2 are the same as before. Only 2 per cent of the light of the bright star remains at deepest phase, though a much larger percentage of its disk remains unobscured. The "equal mass" densities are now 0.08 and 0.04.

The theoretical light-curves computed from these two sets of elements are practically identical. The representation of the observations is shown by the column of residuals in the table of normal magnitudes. The original estimates of brightness were made to the nearest tenth of a magnitude, and consequently the average deviation of a normal point from the computed curve of ±0.05 is entirely satisfactory. The geometrical aspect of the system is shown in Fig. 1. The upper diagram represents the appearance on the assumption of uniform disks, and the lower

[1] Cf. discussion of the orbits of *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei*, *Astrophysical Journal*, **36**, 269, 1912.

[2] *Astrophysical Journal*, **36**, 249–250, 1912.

represents stars completely darkened at the limb. In each case the system is represented at greatest elongation and at principal minimum. The components are relatively distant from each other and no perceptible gravitational elongation in the line of sight is to be expected. Assuming each component to have a mass equal to that of the sun, we may draw the sun to scale in the diagram and thus easily represent the relative diameters and densities. The geometrical relations in the system are practically unchanged if we

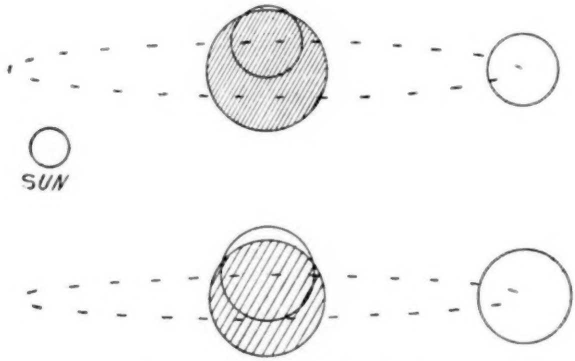


FIG. 1.—The system of *Y Piscium*

adopt other possible values of $1-\lambda_2$, but the effect on relative surface intensities and the color index will be shown later to be very considerable.

My photometric observations of *Y Piscium* are given in the table below. The comparison star is $a = B.D. +7^\circ 5058$, ($9^{mg}.5$). During

PHOTOMETRIC OBSERVATIONS OF *Y PISCIUM*

Date	G.M.T.	Mag.	Date	G.M.T.	Mag.
1912 Sept. 8........	$14^h 55^m$	11.79	1912 Sept. 8.......	$17^h 53^m$	10.01
	15 12	11.90	9........	13 46	8.99
	15 28	12.04		13 58	8.92
	15 54	11.88		14 11	8.91
	16 12	11.62	Oct. 6........	18 50	9.06
	16 37	11.21		19 6	9.00
	16 51	10.99		19 23	8.94
	17 6	10.69	7........	16 30	9.02
	17 36	10.23		16 43	9.17

normal brightness of the variable I found the mean comparison star difference to be $a-v=+1^{mg}.60$. Each point in the table is the mean of 16 measures combined in the customary way so that the systematic errors of the instrument were eliminated and the variable atmospheric extinction minimized. The times are not corrected for the equation of light. The range of the variable is $3^{mg}.05$ with an uncertainty probably not exceeding $0^{mg}.05$. The difference in range, photographic minus visual, is, therefore, $+0^{mg}.35$, and this positive color index is at once an indication of the more advanced spectral type of the faint companion. If, as adopted above, the eclipse is partial, and various assumptions are made regarding the percentage of maximum eclipse, we find:

	Percentage of Light of Bright Star Remaining at Mid-Eclipse, $1-\alpha_0$				
	4.0	3.0	2.0	1.0	0.0
Visual magnitude of fainter star	13.20	12.77	12.47	12.25	12.05
Photographic magnitude of fainter star	15.00	13.64	13.08	12.67	12.40
Color index (relative to primary of spectrum A), $ph-v$	1.8	0.87	0.61	0.42	0.35
$\frac{J_2}{J_1}$(visual) uniform disks	132	85	65	50	41
$\frac{J_2}{J_1}$(visual) darkened disks	80	53	39	31	25

From this table we see that nothing more definite can be said concerning the color index of the faint companion than that it exceeds $0^{mg}.35$; that is, that the star is of a spectral type more advanced than F0.[1] It is not improbable that the color index and the relative surface brightness are much the same as those found below for *RR Draconis*, for which (by the "darkened" solution) $\frac{J_2}{J_1}=42$ and $ph-v=0^{mg}.84$. Moreover, we may find, from a further investigation of this question, that for stars like *Y Piscium* the problem can be worked backward to advantage; for if the color index should ultimately be possible of prediction for whole types of eclipsing stars of great range, the observed difference in range for the deep partial eclipses would enable us to derive highly accurate values of the percentage of the bright star's light not eclipsed, and hence also to obtain precise values of the other constants of the system.

[1] See King's *ph−v* determinations, *Harvard Annals*, **59**, 180.

RR Draconis.—The range of variation of this eclipsing star remained unknown for many years. At minimum light it was invisible in the seven-inch equatorial of the Laws Observatory.[1] On June 20 of the present year, however, I succeeded in following it through minimum light with the 23-inch telescope, and on August 7 Professor Seares followed it during principal eclipse photographically at Mt. Wilson. My observations are given in the table below, each point representing as before 16 comparisons. The

OBSERVATIONS OF *RR DRACONIS*

Date and Epoch	G. Hel. M.T.	Phase	$v-*$	Mag.	Wt.	O.−C.	Remarks
1912 March 16	16^h15^m	$+6^h50^m$	1.04	9.98	3	0.00	Sky good
(866)	16 26	+7 1	1.02	9.96	4	−0.02	
	16 59	+7 34	1.04	9.98	4	0.00	
	17 10	+7 45	1.04	9.98	4	0.00	Getting thick
	17 22	+7 57	1.04	9.98	4	0.00	
	17 36	+8 11	0.92	9.86	2	−0.12	Very thick
April 10	16 7	−4 48	1.12	10.06	2	+0.08	Reading uncertain
(875)	16 19	−4 36	1.06	10.00	4	+0.02	Sky fine
	16 37	−4 18	1.04	9.98	4	0.00	
May 3	14 29	+2 0	0.91	10.76	3	−0.09	Sky poor
(883)	14 45	+2 16	0.70	10.55	4	−0.05	
	14 58	+2 29	0.61	10.46	4	+0.01	
	15 10	+2 41	0.48	10.33	4	+0.03	
	15 36	+3 7	0.20	10.05	4	−0.04	
	15 48	+3 19	0.12	9.97	4	−0.06	
	16 0	+3 31	0.14	9.99	4	0.00	
	16 11	+3 42	0.14	9.99	4	+0.01	
June 20	13 59	−1 34	2.52	11.46	4	+0.04	Sky good
(900)	14 13	−1 20	2.91	11.85	4	+0.05	
	14 25	−1 8	3.16	12.10	4	−0.06	Field high
	14 46	−0 47	3.78	12.72	4	−0.09	
	15 7	−0 26	4.00	12.94	4	0.00	Measures slow but good
	15 36	+0 3	3.97	12.91	4	−0.03	
	16 5	+0 32	4.02	12.96	4	+0.02	
	16 38	+1 5	3.32	12.26	4	0.00	
	16 59	+1 26	2.78	11.72	4	+0.09	

comparison stars were $c = B.D. +62^\circ 1644$, ($8^{mg}.5$), $d = B.D. +62^\circ 1642$, ($9^{mg}.3$), and the maximum light measures give $v-c = +1^{mg}.04$, $v-d = +0^{mg}.13$. The mean times of the observational groups are corrected for the equation of light. The heliocentric date of the minimum observed by me on June 20 is J.D. 2419574.648, G.M.T. It is exactly 900 periods after the initial epoch derived by Seares.[2]

[1] *Laws Observatory Bulletins*, Nos. 6 and 9, 1905, 1907.

[2] *Ibid.*, No. 9, 1907.

The correction, O.−C., to Hartwig is +17 minutes; to the ephemeris from the final elements of *Laws Observatory Bulletin*, No. 9, is −7.3 minutes; and to the ephemeris computed from the elements derived by Seares at Mt. Wilson is +4 minutes. Accordingly, the elements of light-variation I have adopted are:

$$\text{Min.} = \text{J.D. } 2417026.682 + 2^{d}.831073 \text{ E, G.M.T.}$$

and this formula is used to compute the phases in the table of observations. The maximum light of the variable is $9^{mg}.98$, adopted from Seares's determination at the Laws Observatory, and the visual range is $2^{mg}.96$. I have used the visual observations as the basis of a derivation of provisional elements on the limiting hypotheses of uniform and completely darkened disks. The results are as follows:

	Uniform	Darkened
Ratio of stellar radii	0.40	0.58
Radius of bright star	0.099	0.131
Radius of faint star	0.249	0.226
Inclination of orbit	82°.2	85°.7
Least apparent distance of centers	0.136	0.076
Ratio of surface intensities	90:1	42:1
"Equal mass" density of bright star	0.86	0.37
"Equal mass" density of faint star	0.05	0.07
Depth of secondary minimum	$0^{m}.01$	$0^{m}.02$

The principal eclipse is total and lasts for $7^{h}10^{m}$. The light is constant at minimum for $1^{h}20^{m}$, and its value at that time, $12^{mg}.94$, is the visual magnitude of the faint star. The light of the small bright star is 0.934, and of the faint star is 0.066, in terms of the total light of the system. The spectrum of the bright component is classified at Harvard as A5?.[1] The photographic range of variation is $3^{mg}.80$. The color index of $0^{mg}.84$ is that of a G4-type spectrum, provided the bright star is of type A; and corresponds to a G8-type if the primary spectrum is A5.[2] In any case we may safely say that the faint companion is a considerable distance below

[1] Note added February 12, 1913. Professor Pickering writes in a recent letter that Miss Cannon has examined the spectrum of *RR Draconis* again and finds it to be of type A.

[2] King, *op. cit.*

the brighter star in the spectral series. It is quite possible that the visual range would be measured greater or less by another observer, depending on the sensitiveness of the eye to the light the reddish star. The various values of the ranges of some eclipsing stars given by different observers is perhaps to be partly accounted for in this way. Some observers have noted a reddish color of the light at minimum phases, and Blažko[1] has found spectroscopic evidence of a yellow companion to *U Cephei*. The system of *RR Draconis* is represented diagrammatically in Fig. 2 with the "equal

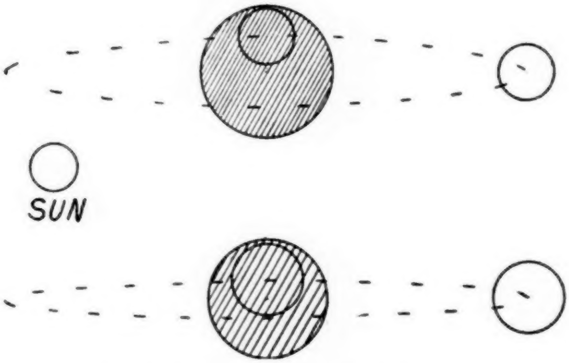


FIG. 2.—The system of *RR Draconis*

mass" sun drawn in to scale. The relatively high density of this system is to be noted.

Although the uncertainty of some of the numerical results obtained for the above stars is somewhat large, the investigation has given a definite indication of the probable outcome of the future study of the spectra of the faintly illuminated, low-density components of certain eclipsing binaries. Preliminary results for other stars, whose ranges of variation are being investigated, confirm the conclusions reached concerning *Y Piscium* and *RR Draconis*.

PRINCETON UNIVERSITY OBSERVATORY
October 28, 1912

[1] *Op. cit.*

DISTRIBUTION OF ENERGY IN THE SPECTRA OF PLATINUM, PALLADIUM, AND TANTALUM

By GEORGE VEST McCAULEY

In an early investigation of the emissive properties of certain solids Paschen[1] concluded that the spectral distribution of energy could be fairly well expressed by the empirical relation

$$E_{\lambda T}=C_1\lambda^{-a}e^{-\frac{C_2}{\lambda T}} \tag{1}$$

for which the wave-length of maximum emission is given by the equation

$$\lambda_m T=\frac{C_2}{a}=\text{constant} \tag{2}$$

where C_1, a, and C_2 are constants depending upon the substance, while T and λ denote absolute temperature of the source and wave-length respectively.

Subsequent attempts to verify this law for metallic radiators have led to widely different results. Lummer and Pringsheim,[2] working with platinum, but only for a very narrow range of temperatures, obtained results that seemed to confirm Paschen's conclusions except that they found the product, $\lambda_m T$, to be only 2620, which is somewhat less than the value obtained by Paschen. Lummer and Kurlbaum,[3] on the other hand, and more recently, W. W. Coblentz,[4] have shown that the exponent "a" is not constant for any one metal, but must be regarded as a function of the temperature if Paschen's law is to express metallic emission. A further result of Coblentz's[5] investigation was that the wave-length of maximum emission, when computed from a single isothermal energy-curve by means of the easily obtained relation,

$$\lambda_m=\frac{\lambda_1\lambda_2\log\frac{\lambda_2}{\lambda_1}}{(\lambda_2-\lambda_1)\log e}, \tag{3}$$

[1] *Weid. Annalen*, **58**, 455, 1896; **60**, 662, 1897.

[2] *Ber. d. Deutsch. phys. Ges.*, **1**, 215, 1899.

[3] *Ibid.*, **17**, 106, 1898.

[4] *Bul. Bur. Standards*, **5**, 339, 1909.

[5] *Phys. Rev.*, **29**, 553, 1909.

previously used by Paschen,[1] was not constant, but increased with increased differences of λ_1 and λ_2. Since equation (3) demands only the general assumption that C_1, a, and C_2 be functions only of the temperature, this result seemed to indicate that even in this more general form Paschen's law was inadequate.

By assuming the empirical law,

$$E_{\lambda T}=C_1\lambda^{-a}\frac{1}{e^{\frac{C_2}{\lambda T}}-1} \tag{4}$$

which is similar to the energy distribution law deduced by Planck[2] for a "black body," and computing the wave-length of maximum emission for a given isothermal curve, Coblentz[3] obtained results that were more nearly constant than with Paschen's form of the law.

In order further to determine the relative agreement of the two assumed laws (1) and (4) with experimental data, the present investigation was undertaken. It was proposed also to compare the emission of the metals studied with that of a "black body," a comparison which had hitherto been impossible because of the lack of an accurate method for measuring the true temperatures of incandescent metal filaments.

In selecting a criterion with which to test the two laws (1) and (4), the writer wished to avoid any that demanded an accurate knowledge of the wave-length of maximum emission. This seemed desirable in view of the difficulty of locating the exact position of this maximum because of atmospheric absorption bands.

The desired end was attained by the following simple solution of equations (1) and (4):

Let E_1, E_2, and E_3 be the emissivities at a given temperature T for the wave-lengths λ_1, λ_2, and λ_3 such that

$$\lambda_3=2\lambda_1=4\lambda_2 .$$

Then since C_1, a, and C_2 are assumed to be functions only of the temperature, equation (1) admits of the solution

$$a=\frac{2\log\frac{E_1}{E_3}+\log\frac{E_1}{E_2}}{\log 2} \tag{5}$$

[1] *Wied. Annalen*, **60**, 665, 1897.

[2] *Vorlesungen über die Theorie der Wärmestrahlung*, Leipzig, 1906.

[3] *Phys. Rev.*, **31**, 317, 1910.

and similarly equation (4) admits of the solution

$$a=\frac{2\log\frac{E_1}{E_3}+\log\left\{\frac{E_1}{E_2}+\frac{2E_3}{E_1}+\sqrt{\left(\frac{E_1}{E_2}\right)^2-\left(\frac{2E_3}{E_1}\right)^2+\frac{4E_3}{E_2}}\right\}}{\log 2}-1 \qquad (6)$$

which should be constant for a given isothermal curve, if the assumed laws are correct.

For comparing the emission of the metals with that of a "black body," the true temperatures of the metal filaments were first measured and the "black-body" emission for all wave-lengths and this same temperature computed by means of Planck's "black-body" distribution law,

$$E_{\lambda T}=C_1\lambda^{-5}\frac{1}{e^{\frac{C_2}{\lambda T}}-1}. \qquad (7)$$

Then by assuming Kirchhoff's law

$$\frac{E_{\lambda T}}{\epsilon_{\lambda T}}=A_{\lambda T}=(1-R_{\lambda T}) \qquad (8)$$

where $E_{\lambda T}$, $A_{\lambda T}$, and $R_{\lambda T}$ represent the emissive, absorptive, and reflecting powers respectively of the metal, while $\epsilon_{\lambda T}$ denotes the corresponding emissivity of a "black body," the reflecting powers of the metals were determined as a function of wave-length and absolute temperature.

METHOD AND APPARATUS

The energy-curves were obtained with the usual spectro-bolometric arrangement shown in Fig. 1. The radiating source at O was inclosed in a suitable vacuum chamber V_1 separated by a rock-salt window W_4 from the large vacuum chamber V_2 which inclosed the entire spectrometer and bolometer. Thus the whole path of the radiant energy was *in vacuo* to eliminate atmospheric absorption as far as possible. The pressure in the spectrometer inclosure was maintained at about 15 mm of mercury by means of a water aspirator.

The spectrometer was of the Wadsworth[1] prism-mirror fixed arm type, furnished for part of the work with a rock-salt prism of

[1] *Phil. Mag.*, **38**, 337, 1894.

59°57′44″ refracting angle, 7-cm refracting edge, and 11-cm base, for the rest with a rock-salt prism of 60°5′35″ refracting angle, 8-cm refracting edge, and 6-cm base.

The slit-width was such as to give an image at the bolometer strip of the same width as the strip itself and was maintained constant for all parts of the spectrum to insure the best possible agreement of measured and actual energy as pointed out by Runge.[1]

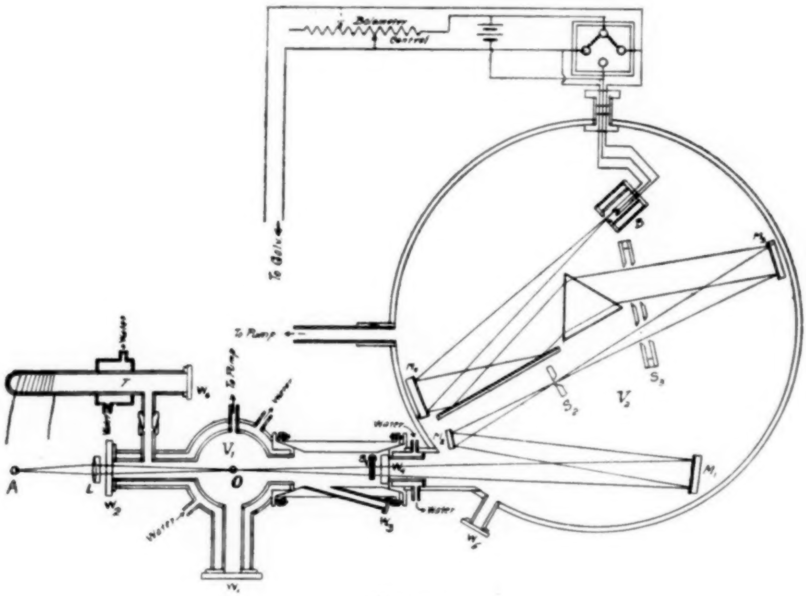


FIG. 1

The galvanometer used was described in a previous paper by Weniger.[2] It had a total resistance of about 7 ohms, a period of approximately 6 sec. with which it was practically aperiodic, and a current sensibility of 4×10^{-10} amperes for a deflection of 1 mm at a distance of 1.5 m. Suitable resistances were placed in series with the galvanometer to reduce its sensibility for work in the various parts of the spectrum.

Two bolometers were employed with strips 12 mm long, blackened with camphor smoke. With the large prism the bolom-

[1] *Zeit. f. Math. und Phys.*, **42**, 205–213, 1897.

[2] *Phys. Rev.*, **31**, 393, 1910.

eter width was 0.2 mm and with the small prism 0.5 mm. The remaining arms of the bolometer bridge were made of No. 22 manganin wire to minimize thermal effects. These coils were wound non-inductively, in the usual way, on a water-filled brass cylinder of about 300 cc capacity and were inclosed in a small wooden box outside the spectrometer vacuum for convenience of adjustment. The bridge balance was controlled by shunting the proper coil with a high resistance, a method recently employed by Abbot and now in general use in this laboratory.

The vacuum chamber V_1 (Fig. 1), in which the filaments were operated, consisted of a water-cooled brass tank of 12.5 cm internal diameter and 20 cm high supported from the large tank V_2 by means of a brass sleeve. The base, to which was attached all the electrical connections and mountings for the filaments, was also water-cooled and detachable. The tank and connecting sleeve were provided with windows W_1, W_2, and W_3 for purposes of calibration and temperature measurements. A steel tube, T, with a water-cooled jacket was fitted to the tank and used for heating calcium[1] to absorb residual oxygen and nitrogen. A triple metal screen S_1 was placed just in front of the rock-salt window W_4 and operated as a shutter through a packed joint in the lower side of the sleeve. All joints were conical and ground to fit, so that a thin film of tallow and beeswax mixture rendered them absolutely tight.

The metal filaments were mounted on heavy copper posts and were capable of being displaced laterally to a slight extent by rotating the base of the tank. This provided for focusing the filaments on the slit and for rotating them to one side during the galvanometer-bolometer calibration to be described later. The copper post clamping the lower end of the filament passed through a packed joint and was provided with an adjusting screw by means of which the filaments could be kept straight at all temperatures.

THE ENERGY-CURVES

The galvanometer deflections were read for successive angles of incidence differing by one minute of arc for a considerable distance on each side of and through the region of maximum emission.

[1] Soddy, *Proc. Roy. Soc.*, **78**, 439, 1906.

For the extreme short and long wave-length portions of the spectrum, the angle of incidence was varied by five minutes of arc. The sensibility of the galvanometer was adjusted, whenever possible, so that the deflections, which were observed to the nearest millimeter, were 10 cm or more. The zero reading was taken before and after each deflection to correct for drift which was small and uniformly steady in one direction during the time necessary to obtain several energy-curves.

The galvanometer deflections were found to be not proportional to the energy falling upon the bolometer strip, which necessitated a proportionality correction determined as follows:

A wide filament carbon lamp, furnished by the General Electric Co., was placed at *A* (Fig. 1) behind the source *O*. An achromatic combination of lenses *L* formed an image of the carbon filament at *O*, the metal filament being rotated now to one side. The prism was then turned $44'$ from the minimum deviation position of the D-line, corresponding to a wave-length of $1.6\,\mu$ or $1.7\,\mu$ depending upon the prism used, and the temperature of the carbon filament raised to $1457°$ C. "black-body" temperature as measured with an optical pyrometer using wave-length $0.658\,\mu$. By means of rotating sectors the radiation from the carbon filament was reduced in intensity in known ratios, and the corresponding deflections of the galvanometer observed. The unit of intensity was arbitrarily chosen as one-twentieth of the intensity of the carbon lamp radiation with no sector interposed. In this way a calibration-curve was obtained connecting galvanometer deflections with units of intensity by means of which it was possible to transform at once from deflection-displacement to intensity-displacement curves.

This calibration was made each time that a series of observations was taken and was made for each sensibility of the galvanometer. Energy-curves thus obtained at widely different times could be compared readily and were not affected by sensibility changes of the bolometer or galvanometer for any given arrangement of the spectrometer.

When using the narrower bolometer strips and corresponding slit-width it was found unnecessary to apply the slit-width correction of Runge (*loc. cit*), the first term of which made less than 1 per

cent change in the intensity in the region of maximum emission where the correction is greatest. With the wider bolometer strips, however, it was necessary to use the first term of the correction.

The dispersion correction for converting prismatic to normal energy-curves was obtained in the usual manner from the dispersion-curve of rock-salt and the computed spectrometer readings for the minimum deviation corresponding to various refractive indices.

The dispersion-curve used in the calibration was obtained from the data[1] of Martin's, Rubens, Rubens and Snow, Langley and Abbot,[2] and was corrected for *vacuo* by means of the dispersion-curve of air determined by Kayser and Runge,[3] extrapolating to long wave-lengths by means of the dispersion formula

$$n = 1.00028817 + 1.316\,\lambda^{-2} + 31600\,\lambda^{-4}$$

which these investigators found would express their results as far as the dispersion had been determined.

The above extrapolation was deemed justifiable in view of the fact that it gave correct wave-lengths for the experimentally located positions of the atmospheric absorption bands at $1.1\,\mu$, $1.4\,\mu$, $1.8\,\mu$, and $2.6\,\mu$ in the solar spectrum and for the CO_2 emission band at $4.4\,\mu$ of the bunsen burner.

The mean wave-length of the D-lines was chosen as the zero of reference; and the emission band at $4.4\,\mu$ from the bunsen burner, together with the sylvite absorption bands at $3.2\,\mu$ and $7.2\,\mu$, were used as checks on the accuracy of the spectrometer adjustments. The zero setting and check were always made first in air, using for this purpose the dispersion-curve of rock-salt with respect to air. Then after evacuating the spectrometer tank, the bunsen burner emission was observed again to check the vacuum correction applied to the dispersion-curve. In each case the band at $4.4\,\mu$ was located to within less than 0.5 per cent of the computed position.

THE "BLACK-BODY" ENERGY-CURVES

The "black-body" emission with which the metallic emission was compared was not measured directly, but was computed by means of Planck's distribution law in the following manner.

[1] Kayser, *Handbuch der Spectroscopie*, **4**, 493.

[2] *Annals of Astrophys. Obs.*, **1**.

[3] *Astron. and Astrophys.*, 428, 1893.

With an arbitrary value of the constant C_1 in the above law, the "black-body" emission was computed for the absolute "black-body" temperature T_b at which the filament was operated and for the particular wave-length, 0.658 μ, at which the "black-body" temperature was measured. Equating this computed "black-body" emission to the observed metallic emission for this same wave-length, by virtue of the definition of "black-body" temperature, a value of C_1 was obtained which would refer the computed "black-body" emission to the same unit as the observed metallic emission. With this new value of the constant C_1 a "black-body" energy-curve was then computed for the absolute true temperature T_t of the metal filament. The inverse ratio of the ordinates of this curve to those of the observed energy-curve determined the reflecting powers according to equation (8).

TEMPERATURE MEASUREMENTS

The metals were obtained in sheet form ranging from 0.1 to 0.2 mm thick. After rolling to about 0.05 mm they were cut into strips 3.5 cm long by 7 mm wide and folded into wedge-shaped filaments with 12° openings. The true temperatures were measured through the window W_1 (Fig. 1) in the manner described by Professor Mendenhall[1] with an optical pyrometer of the Holborn-Kurlbaum type[2] calibrated from the palladium point as measured by Day and Sosman.[3]

In the case of tantalum the "black-body" temperatures were measured simultaneously with the "true" temperatures through the window W_3 by Dr. Forsythe and the writer as explained in the paper[4] referred to above. The relation of "true" to "black-body" temperature for this metal is shown in curve *a* (Fig. 2). In the case of platinum and palladium this relation was obtained from data by Professor Mendenhall,[5] and Waidner and Burgess[6] respectively and is shown in curve *b* (Fig. 2).

[1] *Astrophysical Journal*, **33**, 91, 1911.

[2] C. E. Mendenhall, *Phys. Rev.*, **33**, 74, 1911.

[3] *Amer. Jour. Sci.* (4), **29**, 93, 1910.

[4] *Astrophysical Journal*, **33**, 91, 1911.

[5] *Loc. cit.*

[6] *Bul. Bur. Standards*, **3**, 202, 1907.

EXPERIMENTAL RESULTS

Tantalum.—This metal was obtained from Siemens & Halske, and after rolling was polished with No. 0000 emery paper and washed free from grease and moisture with alcohol. The vacuum in which it was operated was maintained for two days with a Pfeifer rotating mercury pump running continuously, and the calcium was kept at a cherry-red heat for five or six hours prior to

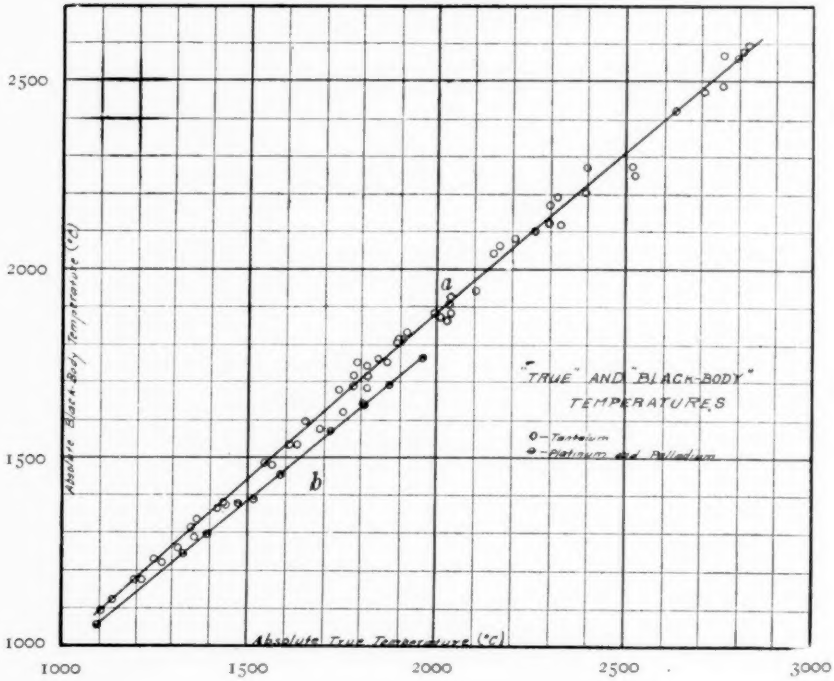


FIG. 2

making observations. These precautions were found necessary because of the great influence of residual gases and grease on this metal at temperatures above 700° C. In the preliminary work it was found that the slightest trace of residual gas would cause the resistance to increase rapidly, and the metal would become brittle, losing its smooth metallic surface. This was also observed by Coblentz[1] and Pirani.[2] With continuous pumping from a per-

[1] *Bul. Bur. Standards*, **5**, 374, 1909. [2] *Ber. d. Deutsch. phys. Ges.*, **5**, 308, 1910.

fectly tight vacuum tank and with heated calcium, it was possible to operate the filaments throughout the entire range of temperatures to within a few degrees of the melting-point of the metal. Even at the very highest temperatures the current and voltage would remain constant, which was considered good working-conditions. After making the observations for five or six isothermal curves, the filaments were found to be ductile with a bright metallic surface as initially, except that they showed the effects of flaking produced by the high temperatures.

The values of "a" computed from equations (5) and (6) were not constant but diminished rapidly at first, reaching an approximately constant value at long wave-lengths. The departure from constancy was more marked in the case of the first of the assumed laws. It must be concluded, therefore, that previous measurements of the temperature variation of this exponent, based upon the assumption that a law of form (1) completely expressed the observed facts, are open to adverse criticism. No consistent variation of "a" with temperature was observed in this instance for a given region of the spectrum. The variation observed with increasing wave-length means nothing more than an erroneous initial assumption regarding the law of energy distribution. Had the assumption demanded the "constants" of (1) and (4) to be functions of wave-length, quite other conditions than (5) and (6) would have resulted for testing the respective laws.

The reflecting powers of tantalum as computed in the manner previously described from a comparison with "black-body" emission are shown in Fig. 3. Curve *a*, shows the values obtained by Coblentz[1] from direct reflection experiments made at room temperature. It is to be observed that the reflecting power diminishes with increasing temperature for wave-lengths greater than $0.7\,\mu$, the decrease being most rapid in the region from $0.8\,\mu$ to $2.0\,\mu$, wherein occurs a minimum reflecting power for high temperatures. For wave-lengths less than $0.7\,\mu$ there seems to be an increase of reflecting power with temperature. This increase was also observed from measurements of the "true" and "black-

[1] *Bul. Bur. Standards*, **7**, 207, 1911.

body" temperature at $0.658\,\mu$ with optical pyrometers.[1] The reflecting powers obtained by this method are shown in Fig. 4, curve *a*.

The variations of the product $\lambda_m T$ with temperature are indicated by Fig. 5. The wave-lengths λ_m were not computed by means

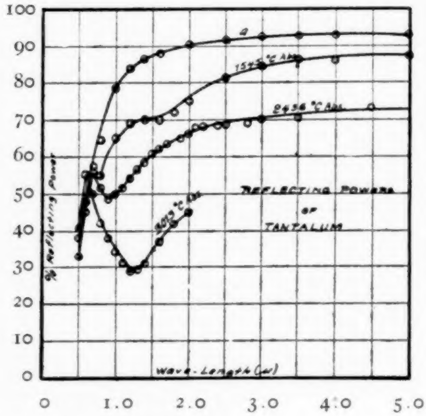


FIG. 3

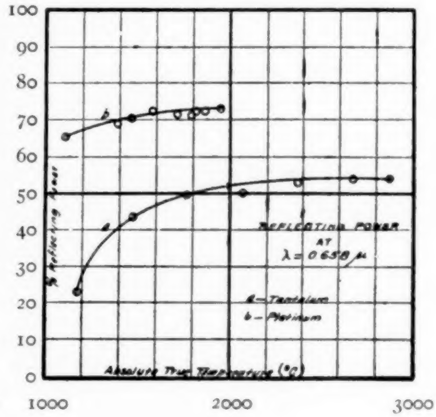


FIG. 4

of any formula resulting from an assumed energy distribution, but were taken directly from the experimental curves. The relation

$$\lambda_m T = \text{const.}$$

is seen at once not to obtain for tantalum. The wave-length of maximum emission shifts much more slowly toward shorter wave-

[1] Let E and E' be the emissivities of the metal from within the wedge and from the outside surface respectively as measured with the pyrometer. Let T_t and T_b be the corresponding "true" and "black-body" temperatures. Then by definition of these temperatures we have by Wien's "black-body" distribution law

$$\frac{E'}{E} = \frac{C_1\lambda^{-5}e^{-\frac{C_2}{\lambda T_b}}}{C_1\lambda^{-5}e^{-\frac{C_2}{\lambda T_t}}} = e^{\frac{C_2}{\lambda}\left(\frac{1}{T_t}-\frac{1}{T_b}\right)}$$

or

$$\log\frac{E'}{E} = \frac{C_2}{\lambda}\left(\frac{1}{T_t}-\frac{1}{T_b}\right)\log e$$

Then by assuming Kirchhoff's law

$$\log\frac{E'}{E} = \log a = \log(1-R) = \frac{C_2}{\lambda}\left(\frac{1}{T_t}-\frac{1}{T_b}\right)\log e$$

from which the reflecting power (R) may be computed as soon as T_t and T_b are known.

lengths with increasing temperature than for a "black body," especially at the higher temperatures. Furthermore, tantalum has its maximum emissivity for low temperatures at a shorter wave-length, and for high temperatures at a longer wave-length than does a "black body."

A few of the spectral energy-curves for tantalum are shown in Fig. 6. They are seen to be perfectly continuous with no bands of

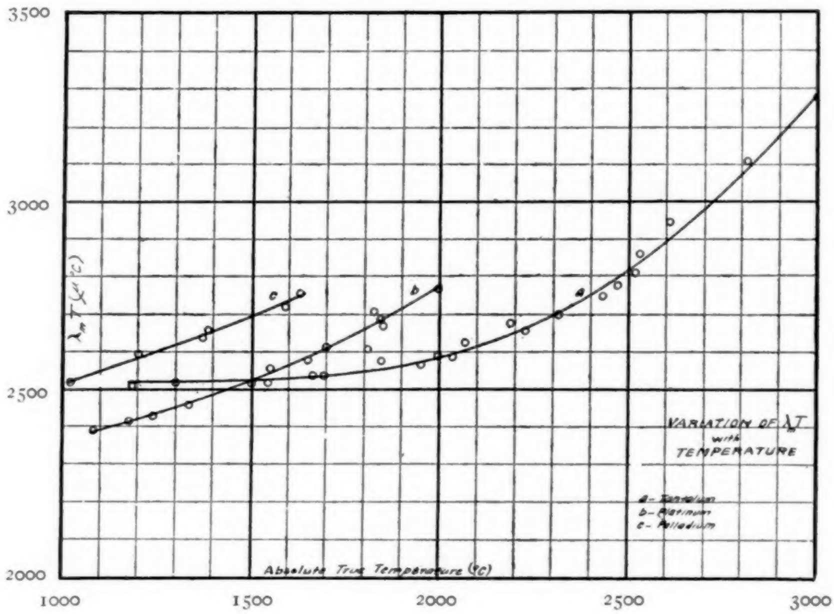


FIG. 5

selective emission. In general form the curves do not differ strikingly from "black-body" energy-curves, of which one is shown in curve *a* (Fig. 6) to the same scale. The emissivity of tantalum, however, diminishes more rapidly than that of the "black body" at the same temperature in the infra-red.

Platinum.—The filaments for this part of the work were cut from platinum obtained from the firm of Baker & Co. They were polished, first with a rounded steel tool on a smooth plate glass, then with No. 0000 emery paper, and cleaned with a dry cloth. They were operated under a pressure of less than 0.1 mm of mercury, merely to eliminate atmospheric absorption as far as possible.

The values of "*a*" computed as for tantalum showed similar variations indicating a better agreement in the case of the second of the assumed laws. Here, also, no consistent variation of this exponent with temperature was observed.

A comparison of the spectral energy-curves with those of a "black body" gave the reflecting powers shown in Fig. 7. Curve *a* was obtained from values observed by Coblentz[1] at room temperature. The same general variations occur here as for tantalum except that the reflecting power in the visible spectrum is more nearly constant for platinum. This constant value in the visible is better illustrated

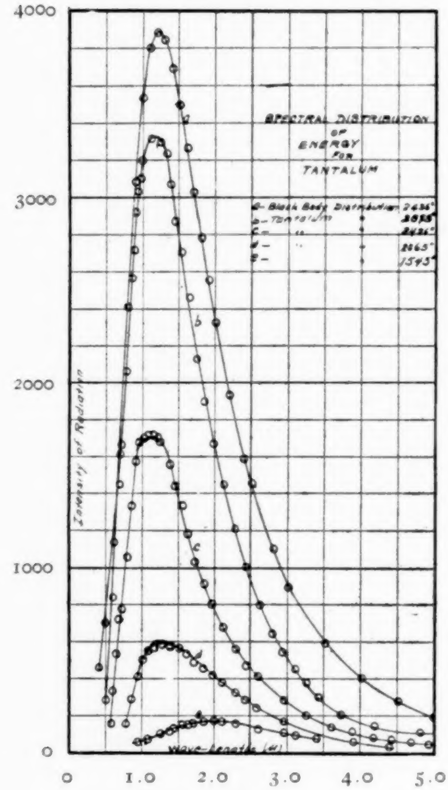


Fig. 6

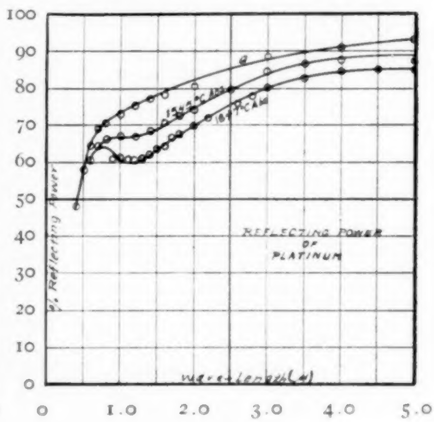


Fig. 7

in curve *b* (Fig. 4) which gives the reflecting powers computed from the relation between "true" and "black-body" temperatures at 0.658 μ as determined by Professor Mendenhall.[2]

The variations of $\lambda_m T$ with temperature are shown in Fig. 5. It will be seen that the same general tendency for this product to

[1] *Bul. Bur. Standards*, **7**, 207, 1911.

[2] *Astrophysical Journal*, **33**, 91, 1911.

increase with temperature obtains here as for tantalum, the increase being more rapid at the higher temperatures.

The spectral distribution-curves are shown in Fig. 8. They are seen to be very similar to those of tantalum, showing a rapid decrease of emission in the infra-red as compared with "black-body" emission shown in curve *a*.

Palladium.—This metal was obtained from Eimer & Amend, and after being rolled to the required thickness was polished with No. 0000 emery paper. The filaments were operated under the same conditions as the platinum. Some difficulty was experienced in keeping the

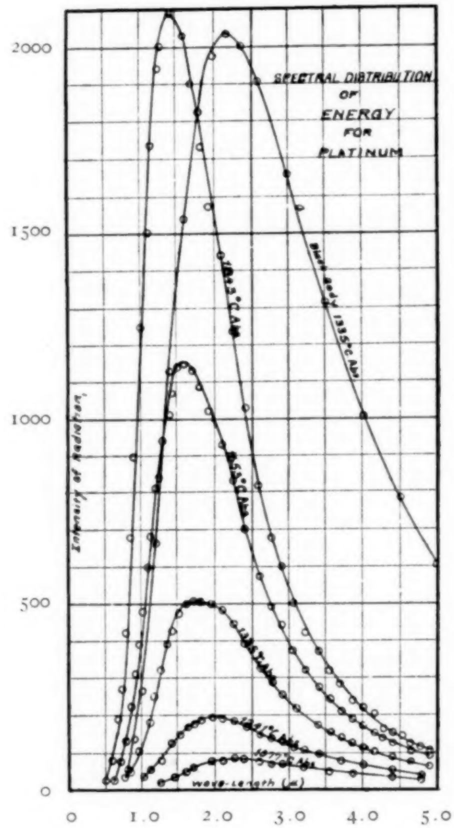


FIG. 8

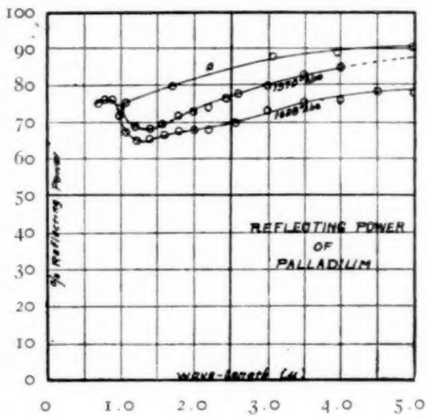


FIG. 9

wedge from opening at the higher temperatures, but by repeated trials it was possible to obtain energy-curves for temperatures ranging from 1000° to 1600° abs.

Owing to the small amount of energy available in the long and short wave-lengths for temperatures at which this metal was operated, it was possible to obtain values of "*a*" through only a narrow region of the spectrum. Even in these narrow limits, however,

the evidence seemed to be in favor of the second of the assumed laws.

The reflecting powers computed as for tantalum and platinum are shown in Fig. 9. Curve *a* gives the values observed by Coblentz.[1]

The dependence of $\lambda_m T$ upon temperature is seen from Fig. 5 and is similar to that of platinum. The values are higher for a given temperature than for either of the other metals, indicating that there is no direct relation between atomic weight and wave-lengths of maximum emission as suggested by Kayser,[2] from the early work of Jacques.

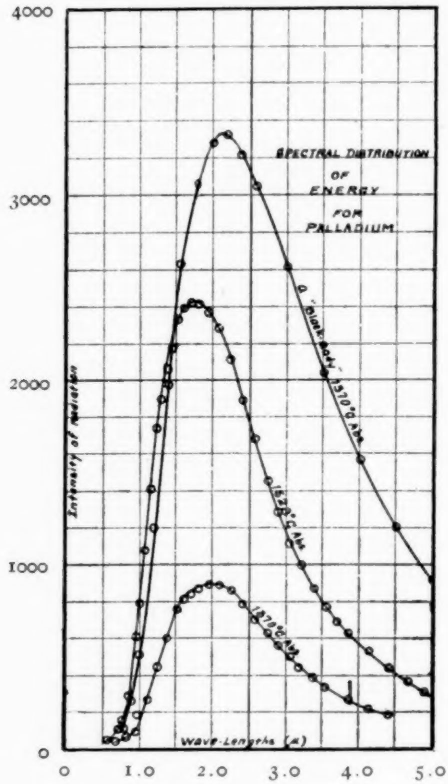


FIG. 10

In Fig. 10 is shown the energy distribution for two temperatures together with that of a "black body."

Owing to the fact that these curves as well as those for platinum were obtained at a time when the humidity of the atmosphere was high, a slight fogging of the rock-salt window separating the two vacuum chambers was practically unavoidable when the smaller chamber was opened to replace the filaments. Consequently the water vapor absorption band at $1.8\,\mu$ shows much stronger in these curves than in the tantalum curves, making the wave-length of maximum emission for some of the curves less certain.

DISCUSSION OF RESULTS

The constant decrease in the values of "*a*" as the longer wave-lengths are considered proves without a doubt the inadequacy

[1] *Bul. Bur. Standards*, **2**, 470, 1906. [2] *Handbuch der Spectroscopie*, **2**, 92.

of the assumed laws (1) and (4). Even for the second of these, which shows the better agreement, the departure from observed values is considerable. Assuming a constant mean value of "a" as determined for a given isothermal curve, the computed emission is in general from four to seven times smaller than the observed values at 6.0μ. The agreement is, of course, better for shorter wave-lengths, but correspondingly worse for longer ones.

Reflecting powers determined in the manner here described are apt to be in error by several per cent, as the "black-body" curves were obtained from a knowledge of the emissivity of the metals at a part of the spectrum where bolometric measurements are somewhat uncertain. The error in the present instance arising from this source was estimated to be from 2 to 10 per cent, being less, of course, at high temperatures. Absolute values only are affected in this way, hence the reflecting powers obtained in this manner show in a qualitative way the relative emissivities of a metal for various wave-lengths and temperatures.

The diminution of reflecting power with increasing temperature for long wave-lengths is in qualitative agreement with the measurements of Hagen and Rubens[1] on the reflecting power of platinum for the residual rays of fluorite as a function of temperature.

The shift of wave-length of maximum emission (Fig. 5) is less rapid for tantalum than for platinum or palladium for low temperatures. Hence temperature estimates[2] of tantalum made from the relation

$$\lambda_m T = 2620,$$

which Lummer and Pringsheim[3] found for platinum, is correct only for a temperature in the neighborhood of 2000° abs.

Fig. 11 shows the agreement of the values of $\lambda_m T$ for platinum with those observed by Paschen.[4] The values obtained by the previous investigator are lower in general than those of the present writer, which, however, are more consistent among themselves. The slight disagreement may be due to the different temperature scales employed or to conduction losses from the thermocouples

[1] *Ann. d. Phys.*, **11**, 888, 1903.

[2] W. W. Coblentz, *Bul. Bur. Standards*, **5**, 375, 1909.

[3] *Loc. cit.*

[4] *Ann. d. Phys.*, **60**, 70, 1897.

used in the earlier work for measuring temperatures. The constant value found by Lummer and Pringsheim seems to have been in error, due possibly to the small temperature range employed, or to lack of polish of the radiating surface. This latter would tend to give a constant value of $\lambda_m T$ as indicated by Pachen's[1] results for such poorly reflecting substances as iron oxide, copper oxide, and lampblack.

SOURCES OF ERROR

The usual errors attendant upon spectro-bolometric measurements from scale readings and subsequent reduction to energy were

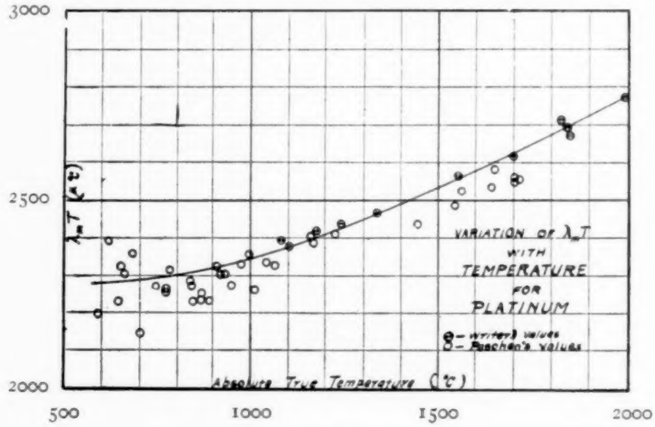


Fig. 11

manifest here. These were minimized as far as possible by employing large deflections, and were estimated to be less than 1 per cent except for the long and short wave-length extremities of the curves. The use of a rock-salt prism further reduced the possibility of error in making the dispersion correction in the region of maximum emission. The point of inflection of the dispersion-curve, at which the dispersion correction is difficult to determine, occurs at about 3.0μ for rock-salt and hence is well beyond the wave-length of maximum emission except for very low temperatures.

Of the constant errors, the reduction of the rock-salt dispersion to *vacuo* is perhaps the most questionable. The CO_2 emission

[1] *Ann d. Phys.*, **58**, 455, 1896; **60**, 663, 1897.

check previously described, however, seemed to justify such a correction. That there are no regions of anomalous dispersion for air as far as 20.0μ is known from determinations of the dispersion of rock-salt, fluorite, sylvite, quartz, and other substances with respect to air for this region of the spectrum. It is possible that the dispersion-curve of air may have a point of inflection in the infra-red, similar to that of rock-salt and fluorite. This would tend to diminish the corrected values of the refractive indices in the dispersion-curve of rock-salt and would thereby increase the corrected wave-lengths in the infra-red. If such a point of inflection does occur in the dispersion-curve for air, it must be beyond 4.4μ or else the departure from the assumed law is small, as the precise location of the CO_2 emission band indicates.

To make sure that no constant error was being introduced by using the bunsen burner emission as a check for the vacuum correction to dispersion and for the zero displacement, it was necessary to show that the position of this band was not altered by the absence of CO_2 in the path of the ray. This was accomplished by observing its position in air containing CO_2 and then in air freed from this gas. Its position was found to be unaltered by the absence of CO_2.

Zero changes of the spectrometer, which are usually very troublesome and almost inevitable, were reduced to a minimum in this instance by the use of iron fittings and mirror supports for the spectrometer. Every part of the spectro-bolometer was carried by heavy iron rods and castings from the base of the spectrometer.

A final source of error is that due to the possible non-black character of the bolometer for certain regions of the spectrum. Any error from this source, however, as shown by Royds,[1] would be of the order of 1 per cent or less beyond 0.8μ and is therefore negligible in this instance.

SUMMARY

The results of the present investigation may be summarized as follows:

1. The distribution of energy in the spectra of tantalum, platinum, and palladium was determined for temperatures ranging from 1000° abs. to the respective melting-points of the metals.

[1] *Phil. Mag.*, **21**, 172, 1911.

2. The assumed metallic radiation laws

$$E_{\lambda T}=C_1\lambda^{-a}e^{-\frac{C_2}{\lambda T}}$$

and

$$E_{\lambda T}=C_1\lambda^{-a}\frac{1}{e^{\frac{C_2}{\lambda T}}-1}$$

were tested by a simple method and were shown to be inadequate to express the emission of the metals.

3. The metals studied were shown to acquire a minimum reflecting power in the early infra-red, becoming more marked at high temperatures.

4. The dependence of reflecting power upon absolute temperature was shown to be similar for all three metals in the infra-red. In the visible spectrum, however, this dependence was found to be different for the different metals.

5. The product $\lambda_m T$ was found to be not constant, but increased rapidly with temperature. For tantalum the absolute value of this product was found to be greater than for a "black body" above 2600° abs.

6. No direct relation was found to exist between atomic weight and wave-length of maximum emission.

In conclusion the writer wishes to express his thanks to Professor Mendenhall for the many helpful suggestions offered and for the special apparatus necessary to carry on the investigation; also to Dr. W. E. Forsythe for his assistance in parts of the work.

University of Wisconsin
August 1911

VACUUM TUBE DISCHARGE IN A MAGNETIC FIELD

By NORTON A. KENT AND ROYAL M. FRYE

While experimenting with a hydrogen vacuum tube placed in a magnetic field, it was found that when the field was created, the visual appearance of the discharge was entirely altered. While it was to be expected that the field would tend to alter the course of the ions, the sweeping nature of the alteration in spectrum, revealed by the direct-vision spectroscope, was a surprise. A search was made through the literature of the subject.[1] The most important results recorded in previous investigations may be briefly summarized as follows:

When a vacuum tube is set vertically in a horizontal magnetic field, there generally results a change in the visual appearance of the tube and also a change in spectrum analogous to that attending the insertion of a Leyden jar in the circuit. The discharge in the field is concentrated in certain parts of the capillary, causing increased resistance; the effect varies with the strength of the field, is most prominent directly between the poles, and is sometimes annulled altogether by increase of temperature. Pressure of the gas, impressed voltage, and kind of glass used modify the result; while reversal of the direction of the discharge (or of the field) produces merely a temporary change. Various gases have been used. With nitrogen Chautard observed that the spectrum developed in the field was the ordinary band spectrum except that the red and orange regions were almost entirely absent. This result he attributed to the change in the effective diameter of the tube. In the case of sulphur, Van Aubel noted a change from a band to a line spectrum. In addition to these spectroscopic investigations, much work has been done upon the influence of the magnetic field upon the resistance of the tube.

In short, in papers previously published, certain writers have hinted at the true cause of the observed changes in spectrum, but, strange as it may seem, not one of them has stated clearly and

[1] See bibliography on p. 189.

fully the substances to which the resulting spectra are in general due. Some observers have noted that the effect is similar to that obtained by the introduction of capacity, resulting in a change from one spectrum of the gas to another of the same gas—which, indeed, is true in some cases. Several have presented the decrease in the effective cross-section of the capillary as sufficient cause for the changes in the spectra. Moreover, attention has been called to the presence of the sodium lines.

As, however, no previous investigator had fully identified the lines of the resultant spectra, a further study of the subject appeared desirable.

APPARATUS

In the experiments performed in this laboratory there were used the following pieces of apparatus:

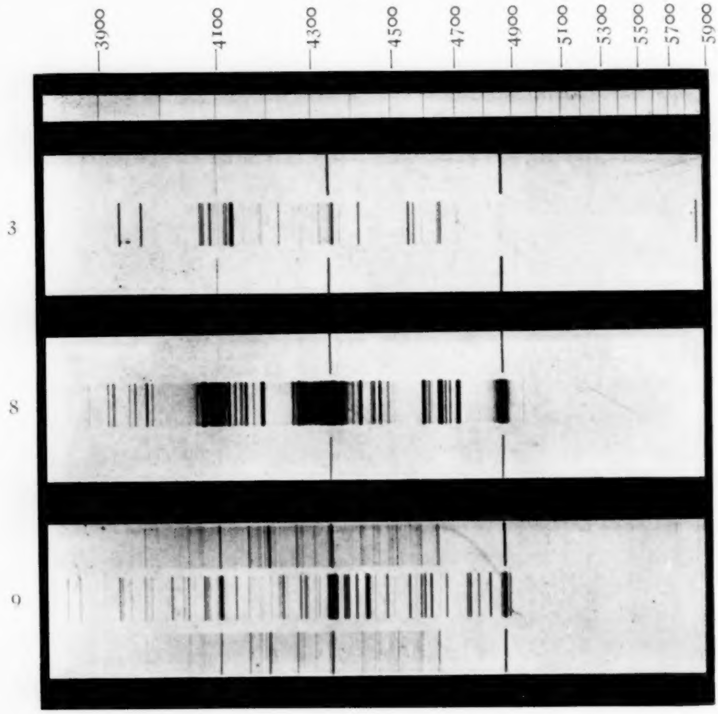
A Weiss electromagnet; diameter of core 10 cm and conical pole tips of 1 cm diameter. The gaps were of 7 or 9 mm. The current throughout the greater part of the work was 16.3 amperes, which corresponded to a field of about 25,000 gausses with the 7 mm gap, and 21,000 with the 9 mm. Field intensities were measured by a bismuth spiral.

A Cox induction coil operated by means of a mercury break. During a large part of the work, two one-quart Leyden jars were employed in parallel with the vacuum tube. Sometimes, however, they were removed and a spark gap placed in series with the tube.

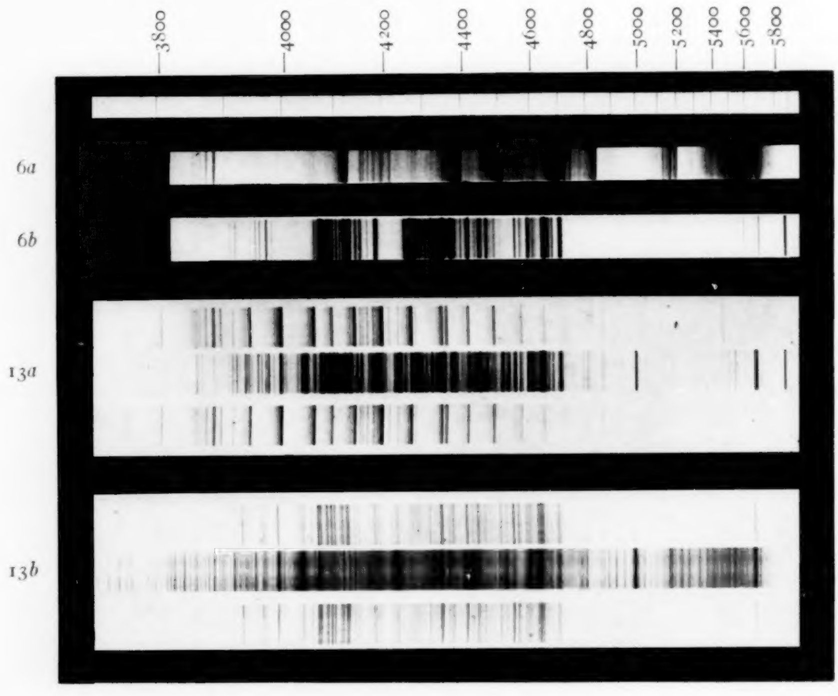
A Hilger single-prism spectrograph, of dispersion about 10 Ångströms per millimeter in the violet and nearly 100 in the red—much too small to obtain the Zeeman effect. By means of a shutter a comparison spectrum could be thrown on both sides of a central region. The plates used were Cramer instantaneous isochromatic.

Tubes of sodium glass, containing oxygen, hydrogen, nitrogen, and carbon monoxide. Further, Dr. Theodore Lyman of Harvard very kindly loaned us some "end-on" tubes containing hydrogen and argon, with impurities. He also constructed for us and filled with hydrogen several tubes of various kinds.

PLATE VII



3900
4100
4300
4500
4700
4900
5100
5300
5500
5700
5900
3
8
9



3800
4000
4200
4400
4600
4800
5000
5200
5400
5600
5800
6a
6b
13a
13b

RESULTS

In this investigation, the record of which was made on forty plates averaging nine exposures each, a few typical gases were employed to illustrate the several effects noted: (*a*) hydrogen as an elementary substance, (*b*) carbon monoxide as a typical compound gas, and (*c*) nitrogen and argon, as good examples of elementary gases having plural spectra. Several combinations of these were also used. Unless otherwise stated, the tubes were set vertically in a horizontal field and the spectrum was photographed from the side. Several photographs, however, showed that it made no difference whether the tubes were viewed "end on" or "side on."

a) The hydrogen tube when not in the field gave the series spectrum alone, and showed no trace of an impurity. In a field of about 25,000 gausses, however, only the F line remained, much weakened and shattered, while there appeared strong lines—chiefly sodium—and weaker ones belonging to the oxygen spark spectrum (see Plate VII, 3). The effect was verified by other tubes. Many tubes became partly shattered between the pole tips and several completely so. These shattered regions were located within the tube at the front and back, as viewed by an eye placed at the slit of the spectroscope. The more completely the discharge was rectified the more unequally shattered were these two regions —an effect to be expected from the heat which must have been generated locally by the sand-blast action of the deflected ions. Viewed vertically through the window of an end-on tube, the discharge appeared to be concentrated in two filaments which hugged the front and rear of the tube and were more unequal in size the more complete the rectification.

The appearance of the sodium lines in the spectrum is thus not at all surprising. However, one would hardly expect the glass to liberate free sodium without, at the same time, liberating an equivalent amount of oxygen in the nascent state. By reason of the fact that oxygen tubes generally soon cease to give the oxygen spectrum, we are confirmed in our belief that this gas is generated continuously while the tube is in the magnetic field. Further, since the entire mass of hydrogen present is not greater than 0.000005 gm, the mass of material from the glass may easily

exceed this, and thus explain the removal of most of the hydrogen lines. The relative strengths of the sodium and oxygen lines may be partially due to conditions which favor the sodium as a carrier, but work on a later tube (carbon monoxide) indicated that the relative number of ions determined the relative strength of the lines. There were in this case twice as many sodium as oxygen ions ($Na_2O = 2\ Na + O$).

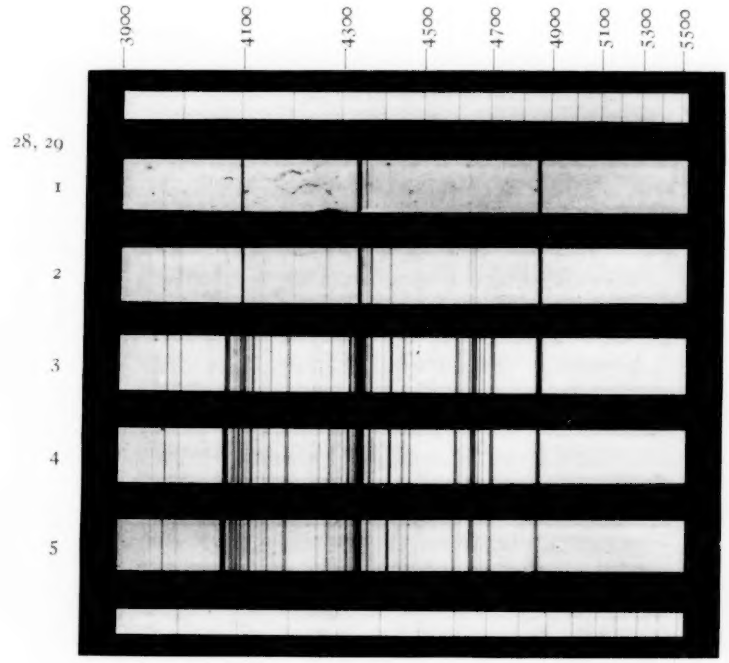
The first effect of gradually increasing the field, using a hydrogen tube of about 1 mm internal diameter, was to broaden the hydrogen lines, then gradually to suppress the weakest, and finally to cause all but the F line to disappear. Meanwhile the oxygen and sodium spectra became visible (see Plate VIII, 28, 29).

By comparing photographs of the spectra of four different sized capillaries approximately 0.2, 0.5, 1, and 3.4 mm internal diameter, in equal fields, the gas being in each case hydrogen, a similar effect was noticed. With the largest capillary the hydrogen lines were simply broadened, but showed no tendency to weaken. Indeed, the result was an actual enhancement (see Plate VII, 8). In the size next smaller the lines were weaker, until, in the two smallest, even the F line had disappeared (see Plate IX, 24). With each decrease in size more sodium lines appeared, and with the smallest capillary there was in addition a considerable region of continuous spectrum. Moreover, in general, whenever the sodium lines are weak, no oxygen lines appear, thus strengthening the hypothesis that the oxygen comes from the glass.

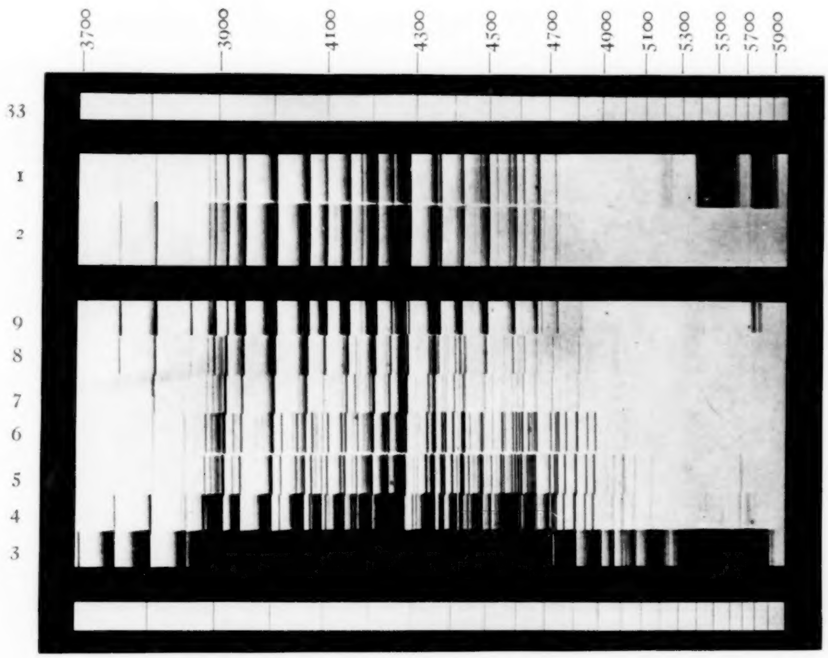
b) In the case of the compound gas, carbon monoxide, with the field off, the regular band spectrum was given, but with the field on, the bands disappeared and were replaced by oxygen and sodium lines, the former here being the stronger (see Plate VII, 6*a*, 6*b*). The preponderance of oxygen may be explained by the decomposition of the carbon monoxide. This recombines, however, when the field is removed, as is shown by the reappearance of the original band spectrum.

Tubes containing hydrogen with a little carbon monoxide as impurity gave combination spectra, as expected (see Plate IX, 22). The oxygen lines were nearly as strong as the sodium. The tubes which became shattered between the poles subsequently gave sodium lines with the field off as well as on.

PLATE VIII



3900
4100
4300
4500
4700
4900
5100
5300
5500
28, 29
1
2
3
4
5



3700
3900
4100
4300
4500
4700
4900
5100
5300
5500
5700
5900
33
1
2
9
8
7
6
5
4
3

PLATE IX

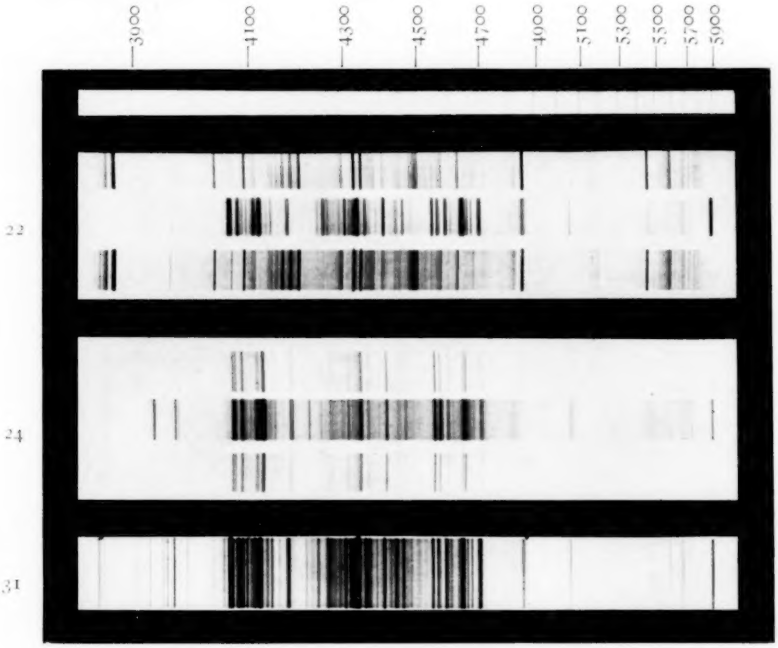


PLATE VII

Three hydrogen tubes: (3) quite pure, (8) containing carbon monoxide and other impurities, and (9) a small amount of argon. The middle spectra show the effect of the field. Notice the enhancement of the hydrogen lines in (8) and (9), in which large-sized capillaries were used; also the manner in which impurities are brought out by the field.

(*6a*, *6b*) Carbon monoxide, field off and on respectively. (13*a*) nitrogen with carbon monoxide as impurity, field off, outside spectrum; on, middle spectrum. In the latter the lines are due to nitrogen and oxygen. (13*b*) shows the coincidence of these lines with the air lines of the iron spark spectrum.

PLATE VIII

(28, 29) Effect of increasing the field in the case of hydrogen. (1) field off, (2) to (5) progressive increase of field. The hydrogen lines at first broaden, then fade out; oxygen and sodium gradually appear. In the third exposure the hydrogen lines have reached the stage shown in a former tube (Plate VII, 8) with larger capillary in a stronger field.

(33) Nitrogen. (1) No field; (2) maximum field. In (2) one of the band systems has disappeared and a few of the stronger lines of the nitrogen line spectrum have come up—the effect noted by Chautard. (3) to (6) inclusive, progressive increase of a weak field through a point where the blue argon spectrum partially replaces the nitrogen band spectrum. Argon will not generally appear in the presence of nitrogen. (7), (8), (9) all with field off—three successive three-minute exposures with a few seconds intervening. Although the intensity of the argon lines immediately falls off, the nitrogen band spectrum reappears but gradually—a unique phenomenon.

PLATE IX

(22) Effect of the field on a tube containing both carbon monoxide and hydrogen. Oxygen and sodium lines are equally strong in the middle spectrum (field on).

(24) Effect of an unusually small capillary in bringing up sodium lines; field on in both cases—central exposure smaller capillary. The same effect is produced in (31) on the larger capillary by an exceptionally strong field.

c) The effect of the field on argon and nitrogen tubes resembled that due to the introduction of capacity. With the creation of the field, the red line spectrum of the argon tube changed to the blue; and the band spectrum of nitrogen was replaced by its line spectrum. Nitrogen, however, behaves in a rather peculiar manner. The observation of Chautard, mentioned above, was confirmed only in the case of a very strong field suddenly applied. Under these circumstances the red and orange parts disappeared from the band system and a few of the spark lines developed (see Plate VIII, 33, 1, 2). But, when a very weak field was progressively increased, the positive band spectrum began to weaken and there appeared a spectrum, most of the lines of which belonged to the blue argon spectrum[1] (see Plate VIII, 3–6). In most cases, the removal of the field was accompanied by an immediate return to the original spectrum; but in this case, three successive three-minute exposures with the field off were marked by only a *gradual* reappearance of the nitrogen band spectrum (see Plate VIII, 7–9).

These changes in the nitrogen spectrum were accompanied by noteworthy visual changes. With no field, the capillary was reddish white, the electrode regions, violet. A small increase of field resulted in no change. At a certain point, however, a violent commotion took place in the tube and the discharge was temporarily nearly choked off. It subsequently became perfectly regular but the color in the capillary changed to an intense blue, while the electrode regions became red, and a green fluorescence appeared at one end of the tube. (It may be mentioned that the creation of the field in nearly every case increased the resistance of the tube enough to render the discharge almost unidirectional.) After the removal of the field, the tube retained its changed appearance for two exposures; then the discharge regained its original color—in agreement with the photographic results. The tube tended to heat more when the field was off than when it was on.[2]

[1] This was an old tube and the manufacturer informs us that it was filled by the "air" method.

[2] The influence of temperature was investigated directly by running a tube in the field first through intervals to keep it as cool as possible and then continuously to produce high temperature. The spectrum showed no change but the discharge was intermittent in the latter case, due probably to increased pressure.

With reference to argon, the following may be noted: A tube evidently containing pure argon (at least no hydrogen lines were visible under ordinary conditions), when subjected to a gradually increasing field, showed at a certain point three hydrogen lines; introducing two one-quart Leyden jars suppressed these lines; and, finally, removal of the jars and a further increase of field resulted in a progressive disappearance—the weakest line first. The above-mentioned conditions appeared to be the only ones under which the remote trace of hydrogen present could be detected.

SUMMARY

The vacuum tube discharge is unquestionably a complex phenomenon, owing to the large number of variables to be considered—potential, current, resistance, temperature, pressure, frequency, and composition of the gaseous mixture contained in the tube. The magnetic field constitutes a new variable and has a decided effect upon those already mentioned. Ordinarily, in a given tube, the size of the capillary is constant, but the magnetic field, by forcing the ions to use only a small part of the capillary, virtually changes the effective cross-section. This increases the resistance of the tube, and diminishes the value of the current flowing. Secondly, the mechanical bombardment of the walls of the tube by the ions liberates material from the glass and adds to the complexity of the gaseous mass under investigation; and, thirdly, these same collisions are probably one of the factors which result in the decomposition or dissociation of the bombarding ions into simpler forms. In general, then, these three factors result in (*a*) the production of the spectra either of substances already present in the tube as impurities, or of dissociation products either (*b*) of the original gas or (*c*) of the glass. As specific examples of each case, we may cite (*a*) the enhancement of the hydrogen lines in an argon tube (which contained hydrogen as an impurity) when this tube was subjected to the proper conditions; (*b*) the change from the band to the line spectrum of nitrogen or the production of the oxygen spectrum in a carbon monoxide tube; and (*c*) the production of not only the sodium but the oxygen spectrum from the

glass. To these new facts may also be added the peculiar phenomena observed with the nitrogen tube.

The writers wish to acknowledge their indebtedness to Dr. Theodore Lyman of Harvard, for loaning old vacuum tubes and filling new ones. We are also indebted to Mr. Charles H. Smith, who assisted us for some days at the beginning of the investigation, and to Mr. A. Herman Wigren, who aided in determining the field strengths employed.

The following are the most important papers dealing with the spectroscopic side of vacuum tube discharge in a magnetic field:

A. de la Rive, *Annales de chimie et de physique* (3), **54**, 238, 1858.
J. Plücker, *Poggendorf's Annalen*, **179**, 88, 151; also **180**, 113, 1858.
A. Trève, *Comptes rendus*, **1**, 36, 1870.
L. Daniel, *ibid.*, **1**, 183, 1870.
P. Secchi, *ibid.*, **1**, 431, 1870.
A. J. Ångström, *ibid.*, **2**, 3369, 1871.
J. Chautard, *ibid.*, **1**, 1123, 1874.
———, *ibid.*, **1**, 1161, 1875.
———, *ibid.*, **2**, 75, 1875.
———, *ibid.*, **1**, 272, 1875.
P. G. Melani, *Nuovo Cimento*, IV, **5**, 329, 1897.
E. Van Aubel, *Journal de physique*, **7**, 408, 1898.
J. Berndt, *Annalen der Physik*, IV, **8**, 625, 1902.
P. Zonta, *Nuovo Cimento*, V, **7**, 321, 1904.
N. Vaccaro, *ibid.*, V, **11**, 222, 1906.
J. E. Purvis, *Proc. Camb. Phil. Soc.*, XIII, **6**, 354, 1906.

PHYSICAL LABORATORY
BOSTON UNIVERSITY
June 1912

EFFECT OF REFLECTION FROM A MOVING MIRROR ON THE VELOCITY OF LIGHT

By A. A. MICHELSON

According to the undulatory theory of light the velocity of light is independent of the velocity of the source, and of the velocity of a mirror at which it is reflected.

According to the emission theory the resultant velocity from a moving source is increased by the component of the velocity of the source. But it appears that different forms of emission theory require different results on reflection from a moving mirror. If the light corpuscles are reflected as projectiles from an elastic wall, then the velocity of light should be increased by twice the component of the velocity of the mirror.[1]

The following arrangement was devised for the purpose of deciding the question experimentally.

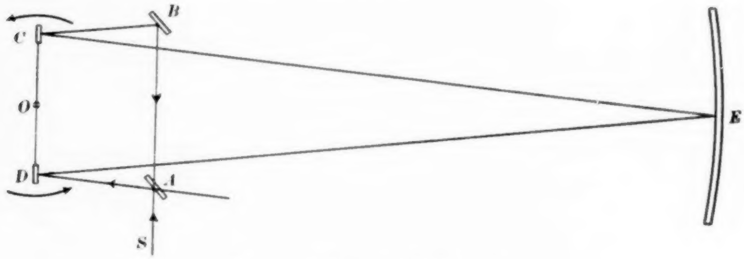


Fig. 1.—Diagram of apparatus

Light from a source at S falls on a lightly silvered mirror A. The reflected pencil goes to a revolving mirror D, thence to the concave mirror E, to the second mirror C revolving about the same axis O, whence it proceeds to the plane mirror B and is reflected back to A. The transmitted pencil pursues the same path in the

[1] An alternative hypothesis, that the velocity of light should be increased by the component of the velocity of the mirror, is suggested by R. C. Tolman but shown to be inadmissible (R. C. Tolman, "Some Emission Theories of Light," *Physical Review*, August 1912).

opposite direction, returning via DA to the starting-point, where it meets the first pencil, producing interference fringes which are observed by means of a telescope with micrometer eyepiece.

According to the undulatory theory the velocity of light is unaffected by the velocity of the mirror while the emission theory[1] requires that

$$\bar{V}=V+rv$$

where $\bar{V}$ is the velocity of light after reflection, V the velocity before reflection and v the component of the velocity of the mirror in the direction of the reflected pencil, and $r=2$ according to the elastic impact theory; while $r=1$ if the mirror surface acts as a new source.

The time occupied by the pencil DEC is

$$T_1=\frac{2(D+d)}{V_1}$$

while that taken by the pencil CED is

$$T_2=\frac{2(D-d)}{V_2}$$

where D is the distance OE, d = distance the revolving mirror moves while light passes over DEC, and V_1 the resultant velocity of the first pencil, V_2 that of the second.

The difference in time is therefore

$$T_1-T_2=2\left[\frac{D+d}{V+rv}-\frac{D-d}{V-rv}\right].$$

But

$$\frac{d}{2D}=\frac{v}{V}$$

whence[2]

$$T_1-T_2=4\frac{D}{V}(2-r)\frac{v}{V}.$$

[1] According to the theory of Ritz (*Annales de chimie et de physique*, Ser. VIII, **13**, 1908) the velocity of light is not affected by reflection from a moving mirror, but is affected by the motion of the source.

[2] Omitting quantities of the second order; v should be replaced by $v \cos \alpha$, but since α is only 3°, the factor $\cos \alpha$ may be taken equal to unity.

The corresponding displacement of the interference fringes is

$$\Delta=\frac{V(T_1-T_2)}{\lambda}=4\frac{D}{\lambda}(2-r)\frac{v}{V}$$

$$\text{For } r=0 \qquad \Delta=8\frac{D}{\lambda}\frac{v}{V}$$

$$\text{" } r=1 \qquad \Delta=4\frac{D}{\lambda}\frac{v}{V}$$

$$\text{" } r=2 \qquad \Delta=0$$

The experiment was tried under the following conditions.

The revolving mirrors were mounted on the shaft of an electric motor the speed of which (measured by a speed counter) could be varied from zero to 1800 revolutions per minute. The distance between centers of the mirrors was $l=26.5$ cm;[1] the distance OE was 608 cm. The light of the carbon arc (in one experiment, sunlight) was filtered through a gelatine film transmitting light of mean wave-length $\lambda=0^{\mu}60$.

The formula for the displacement,

$$\Delta=8\pi n\frac{lD}{\lambda V} \quad \text{if } n \text{ is turns per sec.}$$

or

$$\Delta=\frac{8}{60}\pi N\frac{lD}{\lambda V} \quad \text{if } N \text{ is turns per min.}$$

gives with these data and $r=0$ a displacement of 3.76 fringes for 1000 revolutions per minute.

Following is a table of results of observations reduced to this speed.

It appears therefore that within the limit of error of experiment (say 2 per cent) the velocity of a moving mirror is without influence on the velocity of light reflected from its surface.

Assuming that the effect is actually nil, this interference method may be used to measure the velocity of light with an order of accuracy equal to that of the improved Foucault method or of the "combination" method proposed in an article in the *Phil. Mag.*,

[1] This agrees within less than 1 per cent with the distance calculated from the measured distance AB of the interferometer mirrors.

March 1902. Any one of these three methods is capable of furnishing results of the order of accuracy of one part in one hundred thousand; and differential measurements (e.g., with the light of the

	Δ	Weight
1	3.8	1
2	3.1	1
3	3.2	1
4	4.3	2
5	3.8	2
6	3.93	3
7	3.83	4
	3.81 = weighted mean 3.76 = calculated displacement	

two limbs of the sun) can be obtained with a still higher degree of precision; and thus the effect of a moving source on the velocity of light could be determined.

Ryerson Physical Laboratory
University of Chicago

THE TEMPERATURE OF A WEDGE-SHAPED CAVITY AND ITS USE AS A BLACK BODY

By B. J. SPENCE

Some time ago, in this *Journal*,[1] Professor C. E. Mendenhall described the properties of the radiation emitted by the interior of a wedge-shaped cavity at various temperatures. He found by means of a previously calibrated optical pyrometer that the radiation emitted by such a cavity when the angle of the wedge is small is quite approximately black-body radiation. The simple theory underlying the problem shows that this must be true. By means of the wedge, Professor Mendenhall was also able to compare the temperature of the interior of the wedge with the corresponding black-body temperature of the exterior surface of the wedge, thus gaining a knowledge of the radiation properties of the particular metal of which the wedge was composed. The above statements hold true only so long as the true temperature of the wedge is the same without as within. The results of the experiment warrant the assumption.

If the temperature of the wedge could be determined by means of a thermo-couple inserted somewhere within the wedge, its usefulness would be materially increased. For example, it could be used as a black body for the calibration of optical pyrometers, thus obviating the necessity of the cumbersome and costly black-body furnace. Again, one may determine the relation between the true temperature of a substance and its corresponding black-body temperature, using a pyrometer to determine the black-body temperature. The thermo-couple would indicate the true temperature of the substance.

The following describes, in brief, a method which has satisfactorily measured the temperature of the wedge. A strip of platinum foil about 3.5 cm long and 2 cm wide was wrapped about a piece of very fine bored and thin-walled silica tubing of the same length as the platinum foil, in such a manner that the tube was

[1] **33**, 91, 1911.

completely enveloped and still allowed the platinum foil to form a wedge. The manner of wrapping may be best understood by a reference to the diagram, Fig. 1. The silica tube running the length of the apex of the wedge formed an insulation for the thermo-couple which was threaded through it and retained the junction at the center of the tube.

The wedge was mounted in a brass frame in such a manner that a current could be passed along the length of the wedge. The frame was so constructed that the expansion of the platinum due to heating could be taken up. The thermo-couple was made of two strands of No. 40, B.S. gauge platinum, and platin 10 per cent rhodium and was carefully calibrated, using the melting-points of zinc, antimony, and copper. With couples of wire of larger diameter it was found that the temperature indicated was not the true temperature but a temperature somewhat lower owing to the conduction of the heat away from the junction. In fact this was the reason a wedge 3.5 cm long was used, the attempt being to obtain a place within the tube of uniform temperature distribution and also to minimize the effect of conduction of the thermo-element. Consequently with such an arrangement, it was assumed that the temperature indicated by the couple was the true temperature of the platinum when the heating current was passed through it.

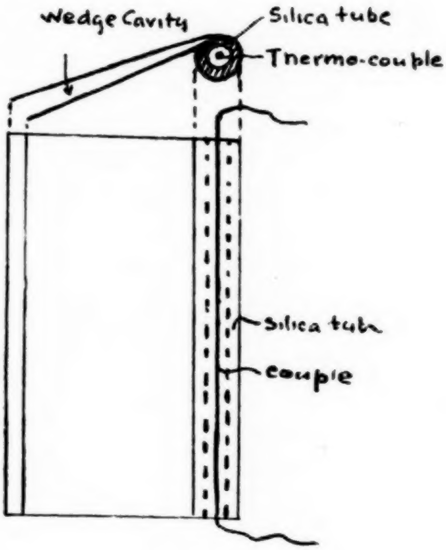


FIG. 1.—Diagram of wedge

Three methods were available for testing the truth of the above assumption. The first method employed the melting of tiny pieces of gold (Kahlbaum) on the outer surface of the wedge. The couple indicated the true temperature of melting gold as accurately as

could be desired. For example, the melting-point of gold taken as 1063° C. gave rise to a thermal electromotive force of 12,140 micro-volts as determined by the standard melting-point calibration. The electromotive force corresponding to the melting of gold on the outside of the wedge was 12,134 micro-volts. This value is an average of 30 determinations of which the maximum deviation of any determination from the mean was not over 60 micro-volts.

The thermal electromotive force corresponding to the melting-point of palladium (Kahlbaum) was also determined. An average value of a large number of determinations indicated a temperature corresponding to 20,700 micro-volts. The value demanded by the extrapolation of the calibration points of the couple was not computed because such an extrapolation is of no value quantitatively.

To further determine the type of radiation from the wedge interior it was viewed with a Holborn-Kurlbaum optical pyrometer which had been previously calibrated by means of a black-body furnace. The current through the pea lamp of the pyrometer corresponding to the melting-point of gold as determined by the furnace calibration was 499 arbitrary units. The current corresponding to the melting-point of gold on the outer surface of the wedge when the interior of the wedge was viewed with the pyrometer was 502 arbitrary units, thus indicating that the radiation from the interior of the wedge approximated more nearly that of the black body at the melting-point of gold than did the black-body furnace.

Finally the difference between the true temperature of the wedge as determined by the thermo-couple and the corresponding black-body temperature indicated by the radiation from the exterior of the wedge was sought for the range from the melting-point of gold to the melting-point of palladium at 1549°.2 C. The black-body temperature was determined with the pyrometer. The results are included in the accompanying table. Column 1 indicates the true temperature determined by the thermo-couple, column 2 the black-body temperature determined by the pyrometer, and column 3 the difference between the true and black-body temperatures.

The values given in the third column agree very closely with the

values obtained by Waidner and Burgess[1] and also those of Professor Mendenhall.[2] Unfortunately no means were available so that the electromotive force of the thermo-couple used in the experiment could have been determined for a temperature corresponding to the palladium melting-point. Had the couple been a standard platinum-platin 10 per cent rhodium couple, a direct

True Temperature	Black-Body Temperature	Difference
985° C.	910° C.	75° C.
1050	960	90
1108	1010	98
1168	1060	108
1227	1110	117
1280	1155	125
1335	1205	130
1390	1250	140
1440	1288	148
1490	1335	155
1530	1365	165

comparison could have been made with the standard couple used by Day and Sosman (*Pub. Carnegie Institution*, No. 157, p. 118). However, the close agreement between the values of the electromotive force at the melting-point of gold and the value given from the standard melting-point determination led one to the conclusion that the electromotive force indicated for the palladium point is very close to the true value of the electromotive force at 1549° C. The difference between the true and apparent temperatures of the platinum for various heating currents also led to the conclusion that the method of measuring the temperature of the wedge with the thermo-couple is a valid one.

UNIVERSITY OF NORTH DAKOTA
November 1912

[1] *Bul. Bur. Stand.*, **3**, 202, 1907. [2] *Loc. cit.*

THE SPECTRA OF SPIRAL NEBULAE AND GLOBULAR STAR CLUSTERS[1]

THIRD PAPER

By E. A. FATH

Since the writing of the second paper[2] a number of additional spectrograms have been obtained with the Mount Wilson 60-inch reflector.

Thê spectrograph used was the same as before except for the last plate, taken in May 1912. For this a new spectrograph, mounted directly in the axis of the telescope, was available. The collimator lens of the new instrument has an effective aperture of 3.6 inches and a focal length of 18 inches. The prism is of UV flint, has an angle of 30 degrees, and will include the full beam. The camera lens has an aperture of 4 inches and a focal length of 8 inches. It is a special lens of portrait type made for the Yerkes Observatory and loaned for this investigation. For this courtesy the writer desires to express his great appreciation. There was no opportunity, at the time, to test the relative speed of the two spectrographs but it seems very probable that the new one has about twice the speed of the other. Lumière "Sigma" plates were used throughout. The method of measurement of the spectrograms was the same as that employed before.

SPIRAL NEBULAE

N.G.C.	Dates of Exposure	Total Exposure
1023.......	1910 Nov. 29, 30, Dec. 1, 2	20^h40^m
1023.......	1911 Oct. 19, 20, 21, 22	38 14
3031.......	Jan. 4, 5, 6	22 39
4594.......	Mar. 29, 30, 31, Apr. 1	17 8
4736.......	May 2	7 40
4826.......	1912 Feb. 10, 11, 12	16 15
5194.......	1911 Apr. 28, 29, 30, May 1	29 20
7331.......	1910 Aug. 27, 28	12 18

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 67.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 49; *Astrophysical Journal*, **33**, 58, 1911.

Owing to a discontinuation of the observations, a description is given of a number of spectrograms which are underexposed, although the information to be derived from such plates is very meager.

As before we shall consider the two classes of objects separately.

N.G.C. 1023.—Absorption lines corresponding to F, 436 $\mu\mu$, G, H, and K are present. The line at 436 $\mu\mu$ may be H_γ, but as it is about 2 $\mu\mu$ to the red of the H_γ position this identification does not appear probable. The grain of the plate having the longer exposure is exceedingly troublesome. The spectrum is very narrow in each case. The general appearance of the spectrum is similar to that of the sun.

N.G.C. 3031.—This is a very good plate. In appearance it exactly resembles the spectrum of a K-type star. There are absorption lines at F, 454, 448, 440 $\mu\mu$, G, groups at 419 and 406 $\mu\mu$, H, and K.

N.G.C. 4594.—This negative is very weak. It is not safe to say more than that the color-curve indicates a spectrum of approximately solar type.

N.G.C. 4736.—As stated in the second paper, there is a difference between the Mount Hamilton[1] and the Mount Wilson plates obtained by the writer, the former being apparently somewhat out of focus while the latter is in good focus and shows absorption lines like the sun. Another plate was taken to check the two preceding. The definition of the third plate is not quite as good as in the first Mount Wilson plate and shows an appearance somewhat like the Mount Hamilton plate, namely, a broadening and slight intensification immediately to the red of H, at 406 $\mu\mu$. Otherwise it agrees fully with the first Mount Wilson plate in showing the prominent solar lines. The latter plate, upon careful examination, also shows the effect at 406 $\mu\mu$ but by no means so prominently as the last. In order to bring out this brighter band it therefore seems necessary to suppress the absorption lines.

N.G.C. 4826.—This plate is very weak. The color-curve is similar to that of a solar-type star.

N.G.C. 5194.—The spectrum is weak in spite of an exposure of nearly 30 hours. The large mirror was in very poor condition at

[1] *Lick Observatory Bulletin*, No. 149.

the time this exposure was made. This doubtless was the cause of the faintness of the spectrum. The color-curve is similar to that of a K-type star. Lines at G and H are certainly present.

N.G.C. 7331.—The plate is underexposed. Lines corresponding to F, G, and H are found. Two other absorption lines are present, one on either side of G, but their position cannot be determined with any accuracy because of the faintness and narrowness of the spectrum. This plate agrees with the Mount Hamilton plate obtained in 1908 which shows lines at G and H.

In the last four years the writer has investigated the spectra of eleven nebulae, *N.G.C.* 224, 650–1, 1023, 1068, 3031, 4594, 4725, 4736, 4826, 5194, and 7331, of which all but 650–1 are certainly spiral. For the most part the spectra are either G- or K-types. In the case of some of the nebulae the spectrum obtained came largely from the nucleus, but this is certainly not the case with the great *Andromeda* nebulae in which a spectrum of solar type was obtained over a strip five minutes of arc in length.

N.G.C. 1068 and 4736.—These are peculiar. The first shows bright nebular lines together with dark lines, although the type of absorption spectrum is not evident. In the case of the second there is increased radiation in the vicinity of 406 $\mu\mu$ near the position of one of the bright lines in some of the Wolf-Rayet stars. These two cases are the only ones in the series of plates secured by the writer at Mount Hamilton and at Mount Wilson which give definite evidence of what may be termed "gaseous" radiation. In a paper[1] published when the present one was practically completed, Professor Max Wolf reports finding emission lines in a number of spiral nebulae, many of which correspond to emission lines in Wolf-Rayet stars. Though the writer's plates show definite emission lines in but two cases, the two series cannot be considered contradictory. Wolf's plates were on about double the scale (spectra 6 mm in length) of those here described, and apparently a finer-grained plate was used.

N.G.C. 5904.—The following absorption lines are present: F, H_γ, G, H, K, and a band at 419 $\mu\mu$ which may possibly be double.

[1] *Sitzungsber. Heidelberger Akad. d. Wiss.*, Abt. A, 1912, No. 15.

The appearance is practically that of a solar spectrum except that H_γ is more prominent here than in the sun.

GLOBULAR CLUSTERS

N.G.C.	Dates of Exposure	Total Exposure
5904.......	1911 May 3, 4, 5	16^h17^m
6093.......	June 1, 2, 3	15 40
6205.......	1912 May 21, 22, 23	15 35
6254.......	1911 May 30, 31	13 05

N.G.C. 6093.—This spectrum is of good density. Absorption lines corresponding to solar F, G, H_δ, H, and K are present. The line corresponding to H_δ is stronger than in the sun. Aside from this the spectrum resembles the solar type. H_γ may be present, for G looks broader than normal.

N.G.C. 6205.—This exposure was made with the second spectrograph as noted above. The exposure of 15^h35^m therefore corresponds to about 31^h when compared with the others. It is the first spectrogram of satisfactory density of this cluster which I have obtained and seems to explain the discrepancy between the first plate obtained at the Lick Observatory[1] and the first Mount Wilson plate.[2] The Lick plate, although faint, indicated stars of various spectral types. These could be picked out because the guiding was such that the cluster was kept quite closely in one position with respect to the slit. The cluster being comparatively coarse, the spectra of but few stars were obtained. On the other hand, for the first Mount Wilson plate, the guiding was such as to allow the cluster to drift slightly in both α and δ. A more nearly average type of spectrum was thus obtained and the F-type was indicated. The plate, however, was not of satisfactory density, so another was taken. In this case a special effort was made to obtain an average spectrum by making the image of the cluster move about over the slit. The spectrum obtained shows absorption lines at H_β, H_γ, G, H_δ, H, and K and a band at 420 $\mu\mu$. H_γ and G appear as a

[1] *Lick Observatory Bulletin*, No. 149.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 49; *Astrophysical Journal*, **33**, 58, 1911.

close double line while H and K are not fully separated. The plate thus indicates either a type of spectrum between F and G, or stars ranging from the one type to the other. If we take the latter, we have approximate agreement among the three plates. A re-examination of the first Mount Wilson plate brought out the fact that the line called H_γ is broader than the others and therefore favors the possibility of its being double as shown in the last plate.

N.G.C. 6254.—This is a comparatively coarse cluster and the spectrum of but one star can certainly be identified. It is of solar type and shows the F, G, H, and K lines. The spectrum of another star is on the plate but it is much fainter than the former, which it appears to resemble. The lines F, H, and K are probably present. G may also be present but the very slight width of the spectrum and the troublesome grain of the plate make this very uncertain.

The four clusters described above agree in showing the solar line G as well as lines of hydrogen and calcium. We thus have three possibilities—the clusters consist of: (1) stars of F and G types, (2) stars of a type intermediate between F and G, possibly F8 in the Harvard classification; (3) a combination of 1 and 2. The latter hypothesis seems most probable.

After the presence of the G line had been recognized in these four cases the spectrograms of the clusters discussed in the second paper were carefully examined again. With the exception of the plate of *N.G.C.* 6205, mentioned above, they gave no evidence of being composed of other than F-type stars.

The number of globular clusters investigated thus numbers twelve, approximately 11 per cent of those of appreciable size and brightness. As stated in the second paper, only the brighter stars of the clusters had any effect on the photographic plate. The result obtained thus far may be stated as follows: *As a whole the brighter stars of the globular clusters investigated have spectra ranging only from the F- to the G-type.* The clusters observed are nearly all that can readily be reached in latitude 34° north and in which the brighter group of stars are fairly condensed and of sufficient brightness to be observed in a reasonable time. Unfortunately these do not include the large group of clusters near 18 hours and

south of -20°. It would be of great interest to know if these, too, show the same small range of spectral type as the northern clusters, and it is hoped that some southern observatory may undertake to answer this question.

SMITH OBSERVATORY
BELOIT, WIS.
November 1912

THE *ALGOL* VARIABLE *RR DRACONIS*[1]

SECOND PAPER

By FREDERICK H. SEARES

The comparison of the photographic results for *RR Draconis* presented in the first paper[2] with the incomplete visual light-curve indicated that the photographic variation is greater than the visual. An examination of the relative dimensions and light-intensities of the two objects composing the system made it practically certain that the difference must be real. This result has since been confirmed by photographs during the minimum of September 10, 1912, made alternately on ordinary and isochromatic plates, the latter being used with a yellow filter. The present paper contains an account of these later observations.

The plates used were Seed "Gilt Edge 27" and Cramer "Instantaneous Isochromatic." The filter is one prepared for the determination of visual magnitudes, and when used in connection with the Cramer plate, the curve of color sensitiveness for spectral types not too far advanced appears to agree satisfactorily with that of the eye. The results derived with this combination will, in what follows, be referred to as photovisual[3] magnitudes.

During the period of variation on September 10, 18 plates, divided equally between the Seed "27" and the Cramer "Iso," were made with the full aperture of the 60-inch reflector. Five one-minute exposures were made on each of the Seed plates, while four exposures of three minutes each were given to all the Cramer plates excepting the last, for which the exposures were two minutes. The total number of exposures is therefore 81. They cover the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 68.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 64; *Astrophysical Journal*, **36**, 368, 1912.

[3] The increasing importance of visual magnitudes determined by photographic methods suggests the desirability of a special designation for results so derived. Though etymologically unsatisfactory, the term *photovisual* is suggested as a convenient notation, the term *visual* being reserved for the indication of results to which hitherto it has generally been applied.

minimum and parts of both branches of the curve, and extend over an interval of 5^h27^m. The atmospheric conditions were satisfactory throughout. The transparency was high, and the steadiness ranged from 3 to 4 on a scale of 10. Further details are given in Tables IX–XI, but before proceeding to their discussion it is necessary to consider the magnitude scale of the comparison stars.

The relative photographic magnitudes of the comparison stars have already been well determined. Referred to an arbitrary zero point, they are shown in the second column of Table IV of the first paper and of Table III below. The results of a provisional determination of the zero point from a plate of duplicate exposure on the field of the variable and on the Pole are given in the last column of Table IV of the first paper. As a control upon this comparison two further photographs of duplicate exposure have since been made on Seed "27" plates. In addition, a series of photographs on Cramer "Iso" plates has been made for the derivation of the photovisual scale used for the reduction of the isochromatic plates exposed during the minimum on September 10. A list of the additional photographs made for scale determinations is given in Table I. Those on "Iso" plates include exposures with the various apertures used for the determination of the relative photovisual magnitudes, and also duplicate exposures on the field of the variable and on the Pole, the latter serving to fix the zero point of the photovisual scale.

Turning first to the revision of the zero point of the photographic scale, we have available plates 902 and 903. The images of the comparison stars for the variable and the Polar Sequence stars shown on these plates were measured with the photometric scale in the usual manner. These exposures were made with a 32-inch diaphragm, and as the distances of the stars from the axis were moderate, we may regard the field as free from aberration. The mean scale readings, uncorrected for distance error, are therefore used for the reduction. These are given in Table II. The numbers and magnitudes of the polar stars are from *Harvard Circular*, No. 170, with the exception of (350) and (362), for which the numbers refer to *Harvard Annals*, **48**, as these objects are not included in the Polar Sequence. Their magnitudes are from an investigation of the Polar Sequence not yet completed, but as the

results for this part of the scale are in close agreement with those of Pickering, they may be used for the present purpose.

TABLE I

PLATES FOR MAGNITUDE SCALE OF COMPARISON STARS

Plate	Date	G.M.T.	No. Exp.	Exp. Time	Aperture	Region	Seeing
	1912						
P 845, "Iso"..	Sept. 15	19^h22^m–19^h33^m	4	3^m	60, 32	Variable	3
846, "Iso"..	15	19 40 –20 0	6	3	60, 32, 14	Variable	3
847, "Iso"..	15	20 5 –20 23	3	3	60	Var., Pole	3–4
856, "Iso"..	16	18 17 –18 32	4	4	60, 32	Variable	5
857, "Iso"..	16	18 41 –19 0	4	4	60, 32	Variable	5
858, "Iso"..	16	19 7 –19 23	3	3	60	Var., Pole	5
900, "Iso"..	Oct. 7	15 40 –16 24	3	10	32	Var., Pole	4
901, "Iso"..	7	16 37 –17 20	3	10	32	Var., Pole	3
902, "27"...	7	17 31 –17 57	3	4	32	Var., Pole	3
903, "27"...	7	18 0 –18 22	3	4	32	Var., Pole	2–3

TABLE II

COMPARISON WITH THE POLE—PHOTOGRAPHIC RESULTS

POLAR SEQUENCE				VARIABLE AND COMPARISON STARS		
H.C. 170		Scale Reading		Star	Scale Reading	
Star	Mag.	902	903		902	903
14	10.52	9.0	8.3	*v*	6.6	6.5
7*r*	10.57	9.4	8.6	1	6.9	6.8
(350)	10.63	9.5	8.8	2	13.1	13.8
5*s*	10.68	9.5	8.9	3	13.7	14.0
16	11.26	11.6	11.5	4	13.7	14.4
19	12.28	14.6	15.6	5	14.5	14.4
7*s*	12.31	14.5	14.7	6	15.1	15.2
20	12.59	14.8	16.4	7	14.8	15.0
(362)	12.78	16.2	17.8	8	15.5	16.5
11*r*	12.83	17.2	18.0	9	16.2	...
23	13.20	18.0	....	10	17.4	16.8
12*r*	13.34	18.5	....	11	17.6	....
24	13.50	18.5	....	..	....	....

The scale readings for the polar stars were plotted against the magnitudes in the second column of Table II, and from the resulting curves were interpolated the magnitudes of the comparison stars and the variable. These results require a correction for extinction.

The zenith distances for the variable for the mean times of exposure of plates 902 and 903 were respectively 48°7 and 52°5.

The corrections are therefore +0.10 and +0.05 mag. The corrected values of the interpolated magnitudes are given in fourth and fifth columns of Table III. The corresponding quantities for plate 792, derived by adding the correction for extinction to the results in the eighth column of Table IV of the former paper, are also given in Table III. The magnitudes of the variable and of star No. 1 from plates 902 and 903 are unreliable, as they represent

TABLE III

PHOTOGRAPHIC MAGNITUDES OF COMPARISON STARS

Star	Prov. Mag.	Interpolated Mag.			Reduction to P.S.			Adopted Mag.	Mag. from P.S.	Diff.
		792	902	903	792	902	903			
v...	0.76	.....	(9.78)	(9.99)	....	(9.02)	(9.23)	9.64	(9.88)	(−24)
1...	0.90	.....	(9.89)	(10.08)	....	(8.99)	(9.18)	9.78	(9.98)	(−20)
2...	3.03	12.07	11.96	12.01	9.04	8.93	8.98	11.91	12.01	−10
3...	3.12	12.27	12.13	12.05	9.15	9.01	8.93	12.00	12.15	−15
4...	3.23	12.18	12.13	12.15	8.95	8.90	8.92	12.11	12.15	− 4
5...	3.40	12.40	12.40	12.15	9.00	9.00	8.75	12.28	12.32	− 4
6...	3.68	12.73	12.58	12.35	9.05	8.90	8.67	12.56	12.55	+ 1
7...	3.70	12.58	12.49	12.32	8.88	8.79	8.62	12.58	12.46	+12
8...	4.02	12.90	12.69	12.65	8.88	8.67	8.63	12.90	12.75	+15
9...	4.03	12.96	12.87	.....	8.93	8.84	....	12.91	12.92	− 1
10...	4.06	12.96	13.19	12.72	8.90	9.13	8.66	12.94	12.96	− 2
11...	4.28	13.00	13.23	.....	8.72	8.95	....	13.16	13.12	+ 4
12...	4.58	.....	.....	.....	....	....	....	13.46	.....	....
13...	4.80	.....	.....	.....	....	....	....	13.68	.....	....
14...	4.89	.....	.....	.....	....	....	....	13.77	.....	....
Means..........					8.95	8.91	8.77			
Adopted mean reduction to Polar Sequence..........						8.88				

an extrapolation of the magnitude-curve defined by the Polar Sequence stars. On this account they are not used in the derivation of the reduction constant, though they may be accepted for a further determination of the relative brightness of the variable and star No. 1. The value previously found was $v-$No. $1=-0.15$ mag. with weight 6. From plates 902 and 903 we derive -0.10 mag. with weight 2. The mean is -0.14 mag. On this account the provisional magnitude of the variable in Table III is 0.76 instead of 0.75, as previously given.

The comparison of the interpolated magnitudes with the pro-

visional results in the second column of Table III gives the values of the reduction constants shown in columns six, seven, and eight. The respective means of 8.95, 8.91, and 8.77 mags. are in satisfactory agreement, and yield as a mean constant for the reduction of the provisional magnitudes to the zero point of the Polar Sequence, 8.88 mags. The final values for the normal brightness of the variable and for the magnitudes of the comparison stars are in column nine of Table III. As a result of the revision, the magnitude of the variable has been decreased by 0.06, the comparison stars by 0.07.

It will be noted that the scale of the adopted magnitudes has been independently derived, and, excepting the zero point, is in nowise based on that of the Polar Sequence. It is of interest, however, to compare the two scales, which is easily done by forming the mean of the interpolated magnitudes for each star. The results are in the column following the adopted magnitudes in Table III. The differences in the last column, with the exception of the first two which are without significance, are all small. There is a suggestion of a small systematic difference, but the interval covered is too short to afford a reliable indication.

Proceeding now to a consideration of the photovisual scale, we have available for the determination the diaphragm plates 845, 846, 856, and 857, all of which were made during normal brightness of the variable. The scale readings for these appear in Table IV. The values in parentheses for star No. 1 are those actually observed, corrected for distance error by means of the average curve ordinarily used for the full aperture of the 60-inch reflector. But, as pointed out in the former paper, these corrections are uncertain in the case of bright stars at any considerable distance from the axis, on account of the large influence of even comparatively small deformations of the mirror. Star No. 1 is of this character, and to avoid the difficulty, the following procedure was followed: The variable during normal light is very nearly of the same brightness as star No. 1 but is situated at the center of the plate. If, therefore, the value of the difference in brightness of the two objects can be independently derived, it will be possible to apply this difference to the scale readings for the variable and derive for star No. 1 values

which will be more reliable than those observed. The required differences may be found from the exposures made with the 32- and 14-inch diaphragms, the results for which are also entered in Table IV. Two additional values are also given by the polar comparison plates, 900 and 901, listed in Table VI, both of which were made with the 32-inch diaphragm. Since the difference

TABLE IV

Mean Readings for Photovisual Scale Plates

Star	845		846			856		857	
	s_{60}	s_{32}	s_{60}	s_{32}	s_{14}	s_{60}	s_{32}	s_{60}	s_{32}
v	6.4	13.6	6.2	13.6	16.4	8.5	13.5	7.5	12.9
1	(7.5)	13.7	(7.4)	13.6	17.3	(7.8)	13.6	(7.5)	13.2
	6.5	....	6.3	....	....	8.6	....	7.6	....
2	13.5	....	13.7	....	....	13.7	....	13.0	....
3	14.5	....	14.2	....	....	14.4	....	13.7	....
4	14.4	....	14.0	....	....	14.1	....	13.6	....
5	11.9	....	11.9	....	....	12.5	....	11.8	....
6	....	....	16.8	....	....	16.1	....	15.3	....
7	14.6	....	15.1	....	....	14.3	....	14.3	....
8	16.7	....	16.8	....	....	15.6	....	15.0	....
9	16.4	....	16.3	....	....	16.2	....	15.3	....
10	....	....	....	....	....	16.5	....	15.7	....

is very small, amounting to only 0.04 or 0.05 mag., we may disregard the small fluctuations in the value of the scale interval from plate to plate and for different parts of the scale. We thus find No. $1-v=+0.14$ scale interval, which, added to the 60-inch exposure values for the variable in the first line of Table IV, gives the required values for star No. 1. These were used in the determination of the magnitude scale instead of those directly observed.

The data in Table IV permit of five separate determinations of the scale of photovisual magnitudes. The exposures were too short for those with the diaphragms to show images for any of the stars excepting the variable and No. 1, and, as these are of the same brightness, we have, in effect, to assume that the photometric scale with which the images were measured is a uniform scale of magnitudes whose constant is to be determined for each plate. Fortunately this is sensibly the case. The resulting mean magnitudes referred to an arbitrary zero point, and the deviations in hundredths

of a magnitude, for each determination of the scale are in Table V. The average deviation, including the effect of relative scale error over a range of three magnitudes, is ±0.045 mag.

TABLE V

PROVISIONAL PHOTOVISUAL MAGNITUDES

Star	Provisional Mag.	Residuals				
		845	846		856	857
		60–32	60–32	60–14	60–32	60–32
v	1.73	−5	−4	+8	+ 2	0
1	1.80	−2	0	0	+ 5	− 2
2	3.76	+3	−3	−4	0	+ 2
3	3.98	−2	+6	+4	− 6	− 1
4	3.92	−5	+5	+4	0	− 4
5	3.29	+1	−1	0	0	− 1
6	4.65	...	+2	−2	− 7	+ 8
7	4.12	+9	−5	−7	+12	− 9
8	4.58	−3	−5	−9	+ 6	+11
9	4.58	+5	+9	+5	−18	+ 1
10	4.80	...	...	...	− 8	+ 8
Average deviation	....	±0.039	±0.040	±0.043	±0.058	±0.043

The zero point of the photovisual scale was derived from the polar comparison plates 847, 858, 900, and 901. The arrangement of Table VI, which contains the scale readings, is similar to that of Table II. It is to be noted, however, that the visual magnitudes of the Polar Sequence stars from *Harvard Circular*, No. 170, are here used instead of the photographic. Precisely the same method has been followed in deriving the reductions to the Pole as was used for the photographic magnitudes. The zenith distances of the variable and the corresponding corrections for extinction were:

	Plate			
	847	858	900	901
Zenith Distance	54°7	48°2	37°5	43°8
Extinction	+0.02	+0.10	+0.19	+0.15

The magnitudes interpolated from the curves defined by the data for the Polar Sequence stars and corrected for extinction are in

columns three to six of Table VII. The provisional magnitudes in the second column are as given in Table V, except that the value for the variable has been revised to include the results from plates

TABLE VI

COMPARISON WITH THE POLE—PHOTOVISUAL RESULTS

Polar Sequence						Variable and Comparison Stars				
H.C. 170		Scale Reading				No.	Scale Reading			
No.	Mag.	847	858	900	901		847	858	900	901
10.......	8.94	....	....	8.7	8.8	*v*	6.3	8.3	12.0	11.8
7*r*......	9.84	....	....	11.6	11.8	1	7.6	8.0	11.7	11.7
5*s*......	9.86	7.5	7.3	11.7	11.8	2	14.2	14.0	16.0	16.1
14.......	10.44	....	....	13.2	13.3	3	14.8	14.5	17.4	17.3
15.......	10.78	....	....	13.7	13.8	4	14.9	14.4	16.7	17.1
16.......	11.07	....	....	14.5	15.0	5	12.6	12.7	14.6	14.7
19.......	12.22	....	15.3	18.3	18.2	7	15.4	15.6	17.8	....
11*r*......	12.24	14.1	14.7	18.2	17.7	8	17.2	17.1	....	....
20.......	12.65	16.4	16.8	....	.....	9	17.4	16.7	....	....

TABLE VII

PHOTOVISUAL MAGNITUDES OF COMPARISON STARS

Star	Provisional Mag.	Interpolated Magnitude				Reduction to Polar Sequence				Adopted Mag.	Mag. from P.S.	Difference
		847	858	900	901	847	858	900	901			
v........	1.76	9.47	10.27	10.26	10.07	7.71	8.51	8.50	8.31	9.98	10.02	− 4
1........	1.80	9.92	10.17	10.15	10.03	8.12	8.37	8.35	8.23	10.02	10.07	− 5
2........	3.76	12.16	12.04	11.68	11.72	8.40	8.28	7.92	7.96	11.98	11.90	+ 8
3........	3.98	12.36	12.19	12.16	12.22	8.38	8.21	8.18	8.24	12.20	12.23	− 3
4........	3.92	12.39	12.16	11.91	12.15	8.47	8.24	7.99	8.23	12.14	12.15	− 1
5........	3.29	11.62	11.74	11.18	11.15	8.33	8.45	7.89	7.86	11.51	11.42	+ 9
6........	4.65	.....	.....	.....	.....	....	....	....	....	12.87	.....	....
7........	4.12	12.53	12.51	12.28	.....	8.41	8.39	8.16	....	12.34	12.44	−10
8........	4.58	12.96	12.88	.....	.....	8.38	8.30	....	....	12.80	12.92	−12
9........	4.58	13.00	12.80	.....	.....	8.42	8.22	....	....	12.80	12.90	−10
10........	4.80	.....	.....	.....	.....	....	....	....	....	13.02	.....	....
Means............................						8.29	8.33	8.14	8.14			
Adopted mean reduction to Polar Sequence.........................						8.22						

900 and 901, which, having been made with the 32-inch diaphragm, are available for the determination of the relative brightness of the variable and star No. 1. The arrangement of Table VII and the

method of reduction to the zero point of the Polar Sequence is the same as that for Table III. As before, the last column gives the difference between the adopted magnitudes resulting from the independent determination of the scale and the means of the interpolated magnitudes which are based upon the Harvard values for the Polar Sequence stars. With the exception of a small constant difference due to the particular distribution of the data used, the agreement between the Harvard visual scale and the photovisual results obtained with the "Iso" plate and yellow filter is satisfactory.

TABLE VIII

Adopted Magnitudes

Star	Photographic Mag.	Photovisual Mag.	Color Index	Spectrum	
				H. Mags.	Mt. W. Mags.
v... (Normal Brightness)	9.64	9.98	−0.34	B4	A2
1	9.78	10.02	−0.24	B6	A4
2	11.91	11.98	−0.07	B9	A8
3	12.00	12.20	−0.20	B6	A5
4	12.11	12.14	−0.03	A0	A9
5	12.28	11.51	+0.77	F8	G6
6	12.56	12.87	−0.31	B4	A2
7	12.58	12.34	+0.24	A6	F5
8	12.90	12.80	+0.10	A3	F1
9	12.91	12.80	+0.11	A3	F1
10	12.94	13.02	−0.08	B9	A8
11	13.16	.....	......	...	...
12	13.46	.....	......	...	...
13	13.68	.....	......	...	...
14	13.77	.....	......	...	...

In Table VIII are collected the final results for the normal brightness of the variable and for the various comparison stars. The fourth column contains the color index, and the fifth, the corresponding spectrum interpolated from the curve of Parkhurst.[1] The apparent preponderance of very early type spectra raises a question as to whether the two scales, if extended to stars of the sixth magnitude, would coincide for stars of the type A0. There is, of course, some uncertainty in the zero points of the adopted scales owing to the small number of comparisons with the Pole. The

[1] "Yerkes Actinometry," *Astrophysical Journal*, **36**, 217, 1912.

average deviation of the separate values of the reduction constants from their means are respectively 0.07 and 0.08 mag. One would not expect, therefore, a relative error greater than a tenth of a magnitude, assuming both the photographic and visual magnitudes used for the Polar Sequence stars to be correct. But an error even of this amount would not be sufficient to account for the deviation of the spectra from the normal distribution.

The results become more satisfactory, if, instead of the Harvard photographic magnitudes for the Polar Sequence stars, we use the values from an investigation with the 60-inch reflector for which preliminary results have recently been obtained. The agreement of the Mount Wilson scale with that of Harvard is excellent for the stars between the tenth and fifteenth magnitudes, but for the brighter stars there is a well-marked divergence amounting to about 7 per cent. If the two scales be brought into coincidence at the sixth magnitude in accordance with the international convention, it appears that the Mount Wilson magnitudes for the fainter stars are fainter than those of Harvard. The difference increases uniformly from the sixth to the tenth magnitude, but from there on is constant and equal to 0.37 mag. Since all of the Polar Sequence stars used above for the determination of the zero point of the photographic scale are fainter than the tenth magnitude (see Table II), the results in the second and fourth columns of Table VIII may be reduced to the Mount Wilson system by the simple addition of 0.37 mag. The spectra corresponding to this modification are given in the last column of Table VIII. It will be observed that the distribution is now normal, and, further, that the spectrum for the variable is now in sensible agreement with the Harvard value A5?.[1] The result is of interest, but not conclusive, for we have practically no reliable information as to the distribution of the spectra of the fainter stars; and, moreover, we are here dealing with only a small group of objects. But as the main purpose of the present discussion is the determination of the relative values of the photographic and the visual variation of *RR Draconis*, the uncertainty is of no immediate consequence.

[1] *Annals Harvard College Observatory*, **56**, 189.

The data relating to the observations during the period of variation are in Tables IX–XI. The scale readings have been corrected for distance error, and those for the variable have been reduced to the mean exposure by the method used for the first series. As before, there is no evidence of systematic difference between the first and last exposures on the same plate. The values of the brightness of the variable in the seventh column of Table XI were interpolated from the magnitude-curves formed with the adopted magnitudes of the comparison stars in Table VIII and the scale readings in Tables IX and X, the argument used being the scale reading in the sixth column of Table XI.

TABLE IX

MEAN SCALE READINGS OF COMPARISON STARS—PHOTOGRAPHIC RESULTS

Star	813	815	817	819	821	823	825	827	829
1	5.0	5.8	5.2	4.6	6.3	6.3	5.6	5.6	5.4
2	12.5	12.0	....	10.7	12.0	11.5	11.6	11.9	11.6
3	12.4	12.2	....	11.4	12.4	12.1	11.9	11.8	11.8
4	12.5	12.4	12.1	11.6	12.8	12.2	12.2	12.0	12.0
5	12.8	12.7	12.1	11.5	12.4	12.1	12.2	12.0	12.1
6	13.5	13.0	12.3	12.3	13.5	13.3	12.7	12.6	12.8
7	13.3	12.9	12.4	12.3	13.3	13.0	12.6	12.5	12.5
8	13.6	13.4	13.1	12.9	14.0	13.5	13.3	13.2	13.4
9	13.7	13.4	13.2	13.2	14.3	13.9	13.3	13.4	13.5
10	13.7	13.7	13.2	13.8	14.1	13.9	13.5	13.5	13.7
11	14.1	13.5	13.1	13.8	14.0	14.0	13.8	13.6	14.1
12	14.9	14.3	14.1	14.6	14.7	14.6	14.5	14.3	14.8
13	15.5	15.3	15.1	15.4	15.6	15.2	14.9	15.0	15.7
14	15.6	15.5	15.3	15.8	15.8	15.9	15.5	15.6	16.2

TABLE X

MEAN SCALE READINGS OF COMPARISON STARS—PHOTOVISUAL RESULTS

Star	814	816	818	820	822	824	826	828	830
1	7.8	7.8	7.8	8.3	9.7	7.8	8.0	7.8	9.1
2	14.0	13.6	13.7	14.0	14.3	13.0	13.6	13.4	15.1
3	14.6	14.2	14.3	14.6	15.3	13.7	14.0	13.8	15.6
4	14.6	14.1	14.2	14.5	15.0	13.6	13.8	13.8	15.4
5	12.9	12.2	12.3	13.1	13.6	11.8	12.3	11.9	13.4
6	16.4	16.2	16.2	15.9	16.6	15.0	15.4	15.5	....
7	15.1	14.7	14.6	15.0	16.0	14.2	14.5	14.4	17.1
8	16.4	16.0	15.9	16.1	17.0	15.0	15.8	15.5	17.8
9	16.4	16.0	15.8	15.8	16.8	14.8	15.2	15.4	17.8
10	17.1	16.6	16.7	16.8	....	15.6	15.7	16.1	....

TABLE XI

MEAN SCALE READINGS AND MAGNITUDES OF *RR Draconis*

1912, September 10

Plate	Exposure	G.M.T.		Phase	*s*	Mag.	O.−C.
			PHOTOGRAPHIC RESULTS				
P 813......	1	$15^h20^m.5$	$0^d.639$	$-0^d.107$	7.4	10.39	+ 4
	2	22.5	0.641	−0.105	7.3	10.36	0
	3	24.5	0.642	−0.104	7.5	10.42	+ 4
	4	26.5	0.644	−0.102	7.1	10.31	−12
	5	28.5	0.645	−0.101	7.8	10.50	+ 6
815......	1	16 5.5	0.671	−0.075	9.2	10.93	− 7
	2	7.5	0.672	−0.074	9.6	11.07	+ 5
	3	9.5	0.674	−0.072	9.7	11.11	+ 2
	4	11.5	0.675	−0.071	9.8	11.15	+ 3
	5	13.5	0.676	−0.070	10.2	11.30	+15
817......	1	16 47.5	0.699	−0.047	11.9	12.34	+ 7
	2	49.5	0.701	−0.045	11.8	12.29	−11
	3	51.5	0.702	−0.044	12.1	12.44	− 1
	4	53.5	0.703	−0.043	12.1	12.44	− 9
	5	55.5	0.705	−0.041	12.6	12.66	+ 2
819......	1	17 22.5	0.724	−0.022	14.8	13.52	+ 6
	2	24.5	0.725	−0.021	14.7	13.50	+ 4
	3	26.5	0.727	−0.019	14.8	13.52	+ 6
	4	28.5	0.728	−0.018	14.7	13.50	+ 4
	5	30.5	0.729	−0.017	14.8	13.52	+ 6
821......	1	18 2.5	0.752	+0.006	15.2	13.52	+ 6
	2	4.5	0.753	+0.007	15.1	13.47	+ 1
	3	6.5	0.755	+0.009	15.0	13.43	− 3
	4	8.5	0.756	+0.010	15.0	13.43	− 3
	5	10.5	0.757	+0.011	15.1	13.47	+ 1
823......	1	18 36.5	0.776	+0.030	14.8	13.49	+ 6
	2	38.5	0.777	+0.031	14.5	13.35	− 7
	3	40.5	0.778	+0.032	14.5	13.35	− 5
	4	42.5	0.780	+0.034	13.8	13.00	−20
	5	44.5	0.781	+0.035	14.2	13.19	+ 5
825......	1	19 15.5	0.803	+0.057	11.2	11.78	+ 6
	2	17.5	0.804	+0.058	11.2	11.78	+14
	3	19.5	0.806	+0.060	11.0	11.69	+13
	4	21.5	0.807	+0.061	10.7	11.57	+ 7
	5	23.5	0.808	+0.062	10.1	11.35	−12
827......	1	19 49.5	0.826	+0.080	8.6	10.88	+ 1
	2	51.5	0.827	+0.081	8.6	10.88	+ 4
	3	53.5	0.828	+0.082	8.5	10.84	+ 2
	4	55.5	0.830	+0.084	8.3	10.76	− 1
	5	57.5	0.831	+0.085	8.2	10.72	− 4
829......	1	20 22.5	0.849	+0.103	7.0	10.46	+ 5
	2	24.5	0.850	+0.104	6.8	10.40	+ 2
	3	26.5	0.852	+0.106	6.8	10.40	+ 4
	4	28.5	0.853	+0.107	6.4	10.26	− 9
	5	30.5	0.854	+0.108	6.6	10.33	− 1

TABLE XI—*Continued*

Plate	Exposure	G.M.T.		Phase	s	Mag.	O.–C.
		PHOTOVISUAL RESULTS					
P 814......	1	15^h41^m5	0^d654	-0^d092	11.4	10.98	0
	2	45.5	0.657	−0.089	11.7	11.07	+ 4
	3	49.5	0.660	−0.086	11.8	11.11	+ 1
	4	53.5	0.662	−0.084	11.7	11.07	− 7
816......	1	16 28.5	0.686	−0.060	13.2	11.85	+ 9
	2	32.5	0.689	−0.057	13.0	11.77	−17
	3	36.5	0.692	−0.054	13.4	11.92	− 5
	4	40.5	0.694	−0.052	13.8	12.04	− 7
818......	1	17 2.5	0.710	−0.036	15.7	12.75	−12
	2	6.5	0.713	−0.033	15.6	12.71	−34
	3	10.5	0.715	−0.031	16.2	12.91	−25
	4	14.5	0.718	−0.028	16.5	13.00	−23
820......	1	17 43.5	0.738	−0.008	17.2	13.25	+ 2
	2	47.5	0.741	−0.005	16.9	13.14	− 9
	3	51.5	0.744	−0.002	17.4	13.32	+ 9
	4	55.5	0.747	+0.001	17.2	13.25	+ 2
822......	1	18 17.5	0.762	+0.016	17.8	13.18	− 5
	2	21.5	0.765	+0.019	17.8	13.18	− 5
	3	25.5	0.768	+0.022	18.1	13.30	+ 7
	4	29.5	0.771	+0.025	17.9	13.21	− 2
824......	1	18 50.5	0.785	+0.039	14.6	12.61	− 6
	2	54.5	0.788	+0.042	14.4	12.53	− 2
	3	58.5	0.791	+0.045	14.2	12.44	+ 4
	4	19 2.5	0.794	+0.048	13.8	12.26	− 2
826......	1	19 31.5	0.813	+0.067	12.7	11.66	− 1
	2	35.5	0.816	+0.070	12.2	11.48	− 4
	3	39.5	0.819	+0.073	12.5	11.59	+ 5
	4	43.5	0.821	+0.075	11.9	11.36	− 1
828......	1	20 4.5	0.836	+0.090	10.7	10.97	− 3
	2	8.5	0.839	+0.093	10.7	10.97	+ 1
	3	12.5	0.842	+0.096	10.3	10.84	− 6
	4	16.5	0.845	+0.099	10.6	10.94	+10
830......	1	20 37.0	0.859	+0.113	10.6	10.50	−13
	2	40.0	0.861	+0.115	11.1	10.66	+ 6
	3	43.0	0.863	+0.117	11.5	10.79	+22
	4	46.0	0.865	+0.119	11.0	10.62	+ 8

Both the photographic and the photovisual light-curves are defined for an interval of about a tenth of a day on either side of the minimum, so that the epoch is reasonably well established by both. The value, 1912, September 10, 17^h53^m5 G.M.T., was given

by both curves. Reduced to the sun, the result is, September 10, $17^h 54^m.5$, or J.D. 2419656.7461 G.M.T. The representation of the minimum by the elements of the first paper is exact. The representation of the visual observations by Lehnert[1] is less satisfactory, however, as each of the three minima observed by him give the correction O.−C.=$+0^d.006$. A deviation in the same direction is also shown by the minimum observed visually by Mr. Harlow Shapley at Princeton in June 1912, and kindly communicated by letter. The correction in this instance is $+0^d.003$.

Both series of photographic observations, and the photovisual results of the latter series were grouped into the normal places shown in Table XII for the formation of mean light-curves. As all of the data thus far accumulated, including the Laws Observatory visual measures, indicate that the light-curve is symmetrical, the results for the two branches were combined in deriving the normal places. The resulting curves are shown in Fig. 1, and their ordinates are given in the second and third columns of Table XIII. The representation of the observations by the curves is indicated by the residuals in the last column of Table XI. The average deviation for the photographic series is ±0.057 mag.; for the photovisual, ±0.077 mag. Omitting plate 818, for which the results are discordant, the average deviation for the photovisual observations becomes the same as for the photographic.

The limits and the amounts of variation, and the extreme color indices are:

	Photographic	Photovisual	Color Index
Maximum	9.64	9.98	−0.34
Minimum	13.46	13.23	+0.23
Amplitude	3.82	3.25	0.57

The two series of photographic observations agree closely in the values given for the photographic range. The photovisual range differs considerably from the visual value of 2.96 mags. found by Shapley on June 21, 1912, with a photometer of the polarizing type. Part of this difference may be due to the lack of red-sensitiveness

[1] *Astronomische Nachrichten*, **191**, 201, 1912.

in the Cramer "Iso" plate, which for stars of more advanced spectral types can scarcely be expected to give exact agreement with normal visual magnitudes. Both series of observations are in agreement, however, in indicating a smaller value for the visual

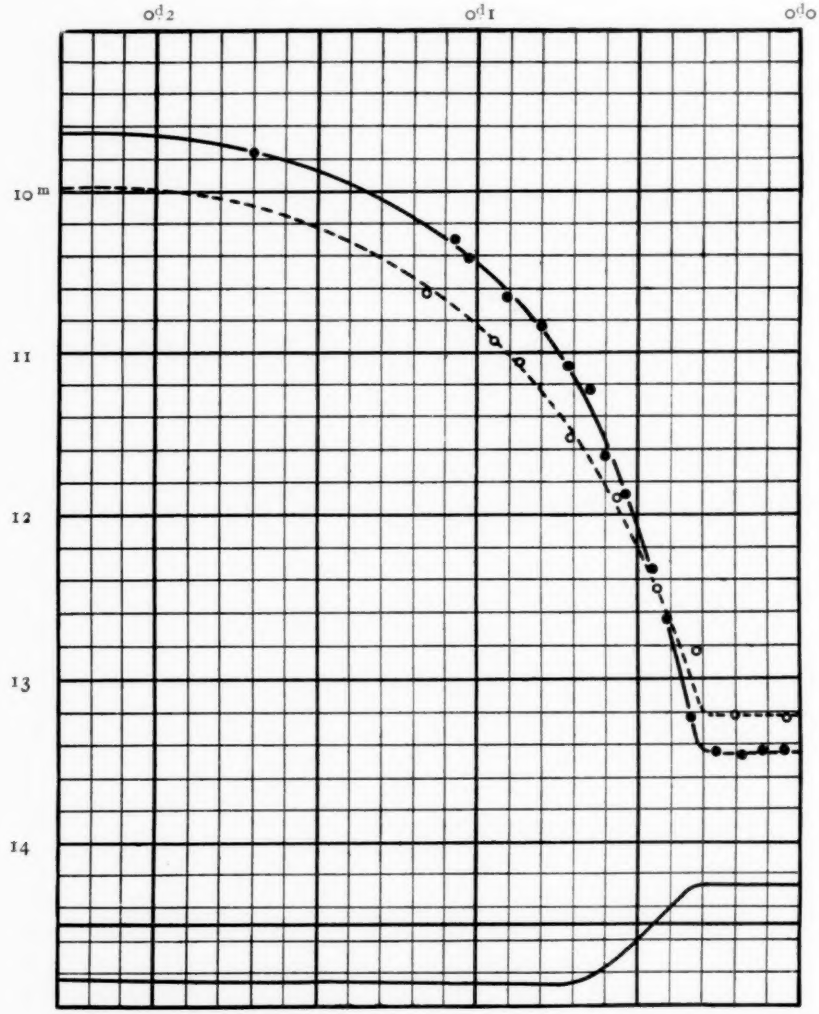


Fig. 1.—Photographic and photovisual light-curves for *RR Draconis*. The latter is the broken curve. The variation in the color index is shown in the lower part of the figure.

range, and there can be no doubt of the reality of the difference in the photographic and visual results.

TABLE XII

NORMAL PLACES FOR MEAN LIGHT-CURVE

Photographic				Photovisual			
Phase	Mag.	O.−C.	No. Obs.	Phase	Mag.	O.−C.	No. Obs.
0^d005....	13.45	− 1	6	0^d004	13.24	+ 1	4
12....	13.45	− 1	7	20	13.22	− 1	4
18....	13.48	+ 2	7	32	12.84	−25	4
26....	13.44	− 2	7	44	12.46	0	4
34....	13.24	0	7	56	11.90	− 6	4
41....	12.64	0	6	71	11.52	+ 2	4
46....	12.33	0	6	87	11.06	0	4
54....	11.88	0	6	95	10.93	+ 2	4
60....	11.65	+ 9	6	116	10.64	+ 5	4
65....	11.24	−10	6	0.220	9.98	0	4
72....	11.08	0	6	.....	.....	....	..
80....	10.84	− 3	7	.....	.....	....	..
91....	10.65	+ 3	7	.....	.....	....	..
103....	10.42	+ 1	7	.....	.....	....	..
107....	10.30	− 4	6	.....	.....	....	..
170....	9.76	0	4	.....	.....	....	..
0.220....	9.64	0	..	.....	.....	....	..

TABLE XIII

MEAN LIGHT-CURVES AND COLOR INDEX OF *RR Draconis*

Phase	Magnitude		Color Index		Phase	Magnitude		Color Index	
	Photo-graphic	Photo-visual	Mt. W.	Laws Obs.		Photo-graphic	Photo-visual	Mt. W.	Laws Obs.
$\equiv 0^d025$..	13.46	13.23	+23	....	0^d090..	10.65	11.00	−35	−33
30..	13.43	13.20	+23	....	100..	10.45	10.81	−36	−34
35..	13.14	12.94	+20	−10	110..	10.30	10.66	−36	−33
40..	12.73	12.63	+10	−13	120..	10.17	10.53	−36	−34
45..	12.40	12.40	0	−17	130..	10.07	10.42	−35	−34
50..	12.08	12.18	−10	−22	140..	9.98	10.31	−33	−33
55..	11.80	12.00	−20	−27	150..	9.89	10.23	−34	−34
60..	11.56	11.82	−26	−21	160..	9.82	10.16	−34	−34
65..	11.34	11.67	−33	−34	170..	9.76	10.10	−34	−34
70..	11.16	11.52	−36	−35	180..	9.71	10.05	−34	−34
75..	11.00	11.37	−37	−36	190..	9.68	10.02	−34	−34
80..	10.87	11.24	−37	−37	200..	9.66	10.00	−34	−34
85..	10.76	11.11	−35	−33	210..	9.65	9.99	−34	−34
0.090..	10.65	11.00	−35	−33	$\equiv 0.220$..	9.64	9.98	−34	−34

The variation in the color index is shown by the curve in the lower part of Fig. 1, and the numerical values are given in the fourth

column of Table XIII. The color index resulting from the comparison of the photographic results with the Laws Observatory visual light-curve in Table I of the first paper is also shown in Table XIII, in the last column. For this comparison the zero point of the visual curve was made to coincide with that of the photovisual results. An examination of the two series of values shows that for phase values greater than $0^{d}.060$ the three curves are practically identical. For values less than $0^{d}.060$ the Laws Observatory curve lies between the other two.

The photographic curve is probably well determined, and the same may be said of the photovisual curve between the phase values $0^{d}.050$ and $0^{d}.125$. The two points on the latter curve within the interval of constant minimum brightness are also probably reliable, but the intermediate portion of the curve is uncertain. It is impossible to improve the representation of the normal point at $0^{d}.032$ without leading to improbable values of the color index. On the other hand, there is no obvious reason why this point should be discordant, for plate 818, upon which it is based, is apparently equal in quality to the others. An examination of the residuals shows that they may be slightly reduced by applying a correction of $+0^{d}.002$ to the adopted epoch, but this raises difficulties with the photographic curve, unless we admit the possibility of a difference in the times of the photographic and photovisual minima. Although the modification of the photovisual epoch would reduce the average residual and lead to a correction for the adopted elements in the direction of that required by the visual observations of Lehnert and Shapley, it seems best to let the results stand as they are for the present.

If the photographic magnitudes be based upon the Mount Wilson system referred to above, it is to be noted that the maximum and minimum limits of variation become 10.01 and 13.83 mags., respectively; and that the extreme color indices are then +0.03 and +0.60. The corresponding spectrum at minimum is F5, and, as the minimum is a total eclipse of the bright body by the larger and darker companion, this value is to be assigned to the fainter object. The corresponding modifications in the photographic mean light-curve and in the color indices in Tables XII

and XIII are made by adding the value 0.37 mag. throughout. The effect upon the curves in Fig. 1 would be to displace the photographic curve downward, and that of the color index upward, by this amount.

But quite independently of any uncertainty as to the spectrum, the investigation shows that the darker component of the system is redder than the brighter by an amount corresponding to 1.3 times the interval separating successive spectral classes; if Shapley's value of the visual range be used, the excess of redness is measured by nearly two spectral classes. The result is one of very great interest, for Shapley, in his investigation of the orbits of eclipsing binaries, has brought forth strong evidence that the darker components of systems of the type of *RR Draconis* are much less dense than their brighter companions, and, hence, presumably mark an earlier stage in stellar evolution. In view of this fact, the exact nature of the spectrum of the darker components of such systems becomes immediately a matter of the greatest interest. Should it appear that we are justified in assigning spectral classes on the basis of the ordinary relations between color index and spectrum, we should arrive at the very significant result that at least some of the so-called advanced spectral types are very probably to be associated with the earlier stages of stellar development. Such a possibility has of course long been recognized, but thus far definite evidence has not been procurable. It is obviously impossible to observe spectroscopically an object as faint as *RR Draconis* at minimum, but there are other *Algol* variables whose examination should prove profitable.

I am greatly indebted to Miss High of the computing division of the observatory for the very careful measurement of the plates upon which the above results are based.

Mount Wilson Solar Observatory
December 16, 1912

Books Received

S. de Ball: *Lehrbuch der sphärischen Astronomie.* Pp. xv+387. Leipzig: W. Engelmann, 1912. M. 20.

James Weir: The Energy System of Matter. Pp. 200, Figs. 12. London: Longmans, Green & Co., 1912. $2.00 net.

T. Van Lohuizen: *Bijdrage tot de Kennis van Lijnen spectra.* Pp. 102, Pls. 2. Den Haag: 1912.

Svante Arrhenius: *Conference sur quelques thèmes choisi de la chimie physique pure et appliqué.* Pp. 112, Figs. 10. Paris: A. Hermann et Fils, 1912. Fr. 3.

Charles Davison: *The Origin of Earthquakes.* Pp. 144, Figs. 26. Cambridge: University Press, 1912. $0.40.

G. A. J. Cole: *Rocks and Their Origins.* Pp. 175, Figs. 20, Cambridge: University Press, 1912. $0.40.

Schott: *Electro-magnetic Radiation.* New York: McGraw-Hill Co., 1912.

Anthony Zeleny and Henry A. Erickson: *A Manual of Physical Measurements.* Pp. 248, Figs. 125. New York: McGraw-Hill Co., 1912.

C. J. P. Cave: *The Structure of the Atmosphere in Clear Weather.* Pp. 144, Figs. 47. Cambridge: University Press, 1912.

H. Kayser: Vol. VI, *Handbuch der Spectroscopie.* Pp. 1065, Pls. 6. Leipzig: S. Hirzel, 1912. M. 60.

Wilhelm Schweydar: *Untersuchungen über die Gegzeiten der festen Erde und die hypothetische Magmaschicht.* Leipzig: B. G. Teubner, 1913. M. 3.50.

Dr. Sigmund Riefler: *Tables of the Weights of Air,* γ_t^b, *of the Air-Pressure Equivalents* β_t^b, *and of Gravity, g.* Pp. 102. Berlin: Julius Springer, 1912. M. 6.

Frank Harris: *Gravitation.* Pp. 108, Figs. 17. New York: Longmans, Green & Co., 1913. $1.00.

E. Rutherford: *Radioactive Substances and Their Radiations.* Pp. 699, Pls. 8, Figs. 123. Cambridge: University Press. 15*s.* net.

Max Wolf: *Stereoskopbilder vom Sternhimmel.* Leipzig: Verlag vom Johann Ambrosius Barth, 1913. M. 5.

Resultados del Observatorio Nacional Argentino en Córdoba, Vol. 20, Catálogo de 5791 Estrellas. Cordoba: Publicado por el Observatorio, 1911.

Annuaire pour l'an 1913, publié par le Bureau des Longitudes. Paris, Gauthier-Villars. Contains statistics relating to astronomy, geography, weights and measures and meteorology (the physical and chemical tables appearing in the alternate volumes).

Annales de l'Université de Lyon, Nouvelle Série; I, Fascicule 32, 1912; contains, 1, Observations of the system of Jupiter by Guillaume; 2, Meridian observation of Struve double stars by Merlin and Flajolet; 3, Micrometric observations of Struve double stars by Cadet, Lagrula, and Guillaume.

Fascicule 33 by Luizet is in two parts, 1, "Les Céphéides considérées comme étoiles doubles"; 2, "De l'étoile variable δ *Céphée*."

H. G. Zeuthen: ***Die Mathematik im Altertum und im Mittelalter.*** ("Die Kultur der Gegenwart." Herausgegeben von Paul Hinneberg.) Die mathematischen Wissenschaften, unter Leitung von F. Klein. Des Gesamtwerkes, Teil III, Abteilung I. Erste Lieferung. Pp. 95, Figs. 20. Leipzig: B. G. Teubner, 1912. M. 3.

Thomas Edward Heath: *Tracks of the Sun and Stars, A.D. 1900 to A.D. 37900:* Shown in Space of Three Dimensions and a Description of the Method Used for Drawing Them. Pp. 17+vi, Figs. 3, Charts 6. Sold by W. Wesley & Son, London, and by Flatters & Garnett, Manchester, 5*s*. net.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

Where unusual expense is involved in the publication of an article, either on account of length, tabular matter, or illustrations, arrangements are made whereby the expense is shared by the author or by the institution he represents.

Authors will please carefully follow the style of this JOURNAL in regard to footnotes and references to journals and society publications.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

If a request is sent *with the manuscript,* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

The editors do not hold themselves responsible for opinions expressed by contributors.

The ASTROPHYSICAL JOURNAL is published during each month except February and August. The annual subscription price is $5.00; postage on foreign subscriptions, 62 cents additional. Business communications should be addressed to the *University of Chicago Press, Chicago, Ill.*

All papers for publication and correspondence relating to contributions should be addressed to *Editors of the* ASTROPHYSICAL JOURNAL, the *University of Chicago, Chicago, Illinois, U.S.A.*